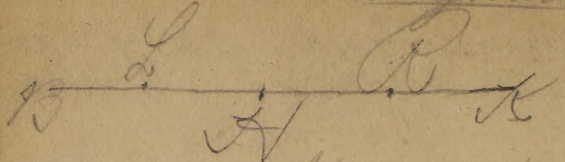
Was there room for
your feet yesterday
under the table.
Grandma?

Geometry.

Etta F. Barton.
742 Centre St.
Jamaica Plain.
Mass.

First Original



Hyp. The line BK divided into BA and AK at pt. A and BA bisected at L forming BL = LA and AK bisected at R forming LR = RK.

Prove.

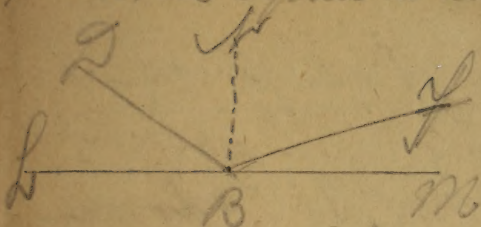
$\frac{1}{2}BK = LR$

1) $LA = LB$ } (Hyp.)
2) $AR = RK$ }
3) $LA + AR = LB + RK$ (Ax. 1)
4) $LA + AR = LR$ (Ax. 7)
5) $LB + RK = LR$ (Ax. 8)
6) $LA + AB + LB + RK = 2\,LR$ (Ax. 1)
7) $LB + LA + AR + RK = BK$ (Ax. 7)
8) $BK = 2\,LR$ (Ax. 8)
9) $\frac{1}{2}BK = LR$ (Ax. 4)

∴ If a line be divided into any no. parts and each of these parts be bisected, the distance between the bisecting points will be half the original line. Q. E. D.

Theorem I.

The sum of all the angles formed at a pt. in a straight line on one side of the line in the same plane, is equal to two right angles.



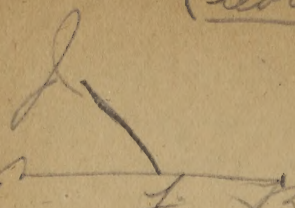
Prove $\angle LBD + \angle DBY + \angle YBM = 2Rt\angle$

Const. Erect $\perp BA$ at pt. B.

1) $\angle LBA = 1Rt\angle$ (Def of Rt $\angle$)
2) $\angle ABM = 1Rt\angle$ (" " ")
3) $\angle LBA + \angle ABM = 2Rt\angle$ (Ax 1)
4) $\angle LBD + \angle DBA = \angle LBA$ (Ax 7)
5) $\angle ABY + \angle MBY = \angle ABM$ (Ax 7)
6) $\angle LBD + \angle DBA + \angle ABY + \angle MBY = 2Rt\angle$ (subs 2)
7) $\angle DBA + \angle ABY = \angle DBY$ (Ax 7)
8) $\angle LBD + \angle DBY + \angle YBM = 2Rt\angle$ (subs)

Q. E. D.

(Cor. I to Th. I.)



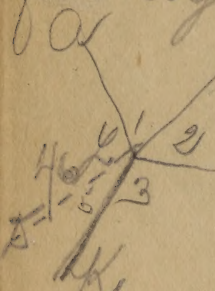
Prove $\angle FE$ is supp of $\angle FK$.

1) Sup of $\angle FB = 2Ra - \angle FB$ (Def of Sup)
2) $\angle EF + \angle FB = 2Ra$ (Th. 1)
3) $\angle EF = 2Ra - \angle FB$ (Ax 2)
4) $\angle EF$ = Sup of $\angle FB$ (Ax 8 on Eq 1)

Q.E.D.

Cor. II to Th. I.

The sum of all the angles formed about a point in a plane is equal to four right angles.



Hyp $\angle 1, \angle 2, \angle 3, \& \angle 4$ formed about pt. O in a plane

Prove $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 4Ra$.

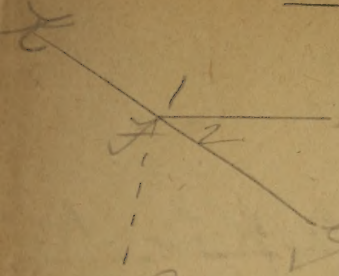
Cons Prolong BO to K (Post. 2)

1) $\angle 6 + \angle 1 = 2Ra$ (Th. I)
2) $\angle 5 + \angle 3 + \angle 2 = 2Ra$ (Th. I)
3) $\angle 6 + \angle 1 + \angle 5 + \angle 3 + \angle 2 = 4Ra$ (Ax 1)
4) $\angle 6 + \angle 5 = \angle 4$ (Ax 7)
5) $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 4Ra$ (subs)

Q.E.D.

8 T 9.

Th II.



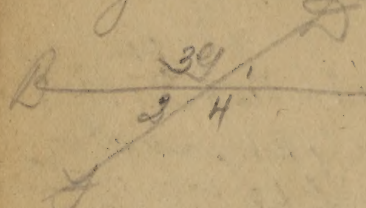
⊙ Hyp. St. lines E F + F G
on opp. sides of ⊙ F.
meeting appt. F in ⊙ F
making ∠1 + ∠2 = 2 R.A.

Prove K F + F G form a St. Line
i.e. F G is a prolongation of K F

If K F + F G do not form a St. line, it is possible to draw from F a line that will be a continuation of K F (Post 2) Let F P be that line
Then ∠K F P + ∠P F G form a St. line
1) ∠1 + ∠C F P = 2 R.A. (Ax. I.)
but 2) ∠1 + ∠2 = 2 R.A. (Hyp)
3) ∠1 + ∠C F P = ∠1 + ∠2 (Ax 8)
∴ 4) ∠C F P = ∠2 (Ax 2.)
but 5) ∠C F P > ∠2 (Ax 6)
Therefore K F + F P cannot form a st. line since supposing that they do, has brought us to a wrong conclusion: and since P represents any line drawn from F as a prolongation of K F except F G, it follows K F + F G is a St. L

Theorem III.

If two straight lines cut each other the opposite or vertical angles are equal.



Hyp. The straight lines B C & D D cutting each other at pt. G forming opposite angles $\angle 1, 2$ & $3, 4$.

Prove $\angle 1 = \angle 2$
$\angle 3 = \angle 4$

---

1) $\angle 1 + \angle 3 = 2$ R.A. (Th I)
2) $\angle 2 + \angle 3 = 2$ R.A. (Th I)
3) $\angle 1 + \angle 3 = \angle 2 + \angle 3$ (Ax 8)
4) $\angle 1 = \angle 2$ (Ax 2)

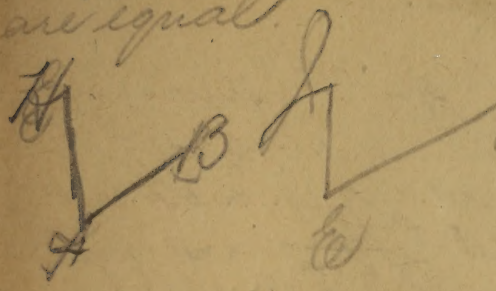
2 E.D.

1) $\angle 3 + \angle 2 = 2$ R.A. (Th I)
2) $\angle 4 + \angle 2 = 2$ R.A. (Th I)
3) $\angle 3 + \angle 2 = \angle 4 + \angle 2$ (Ax 8)
4) $\angle 3 = \angle 4$ (Ax 2)

2 E.D.

Theorem IV.

Angles whose sides have the same or opposite directions from their vertices are equal.



Hyp. Sides AZ + BZ have the same (or opp) direction from vertex Z, that is, E + RE have from Z, or E.

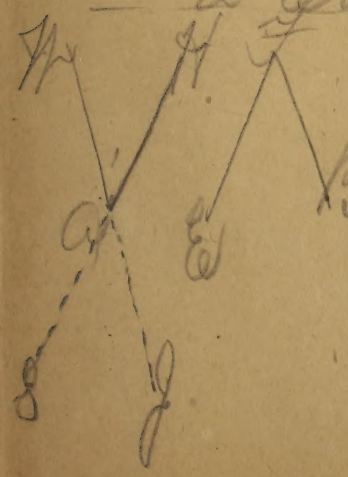
Prove. $\angle Z = \angle E$.

1) AZ has same direction as $\left.\begin{matrix} E \\ RE \end{matrix}\right\}$ (Hyp)
2) BZ " " " "
3) ∴ The difference in direction between AZ + ZB that is between E + ER
4) $\angle Z = \angle E$ (Con. to Def of R. A.)

Q. E. D.

Part II.

Hyp. same



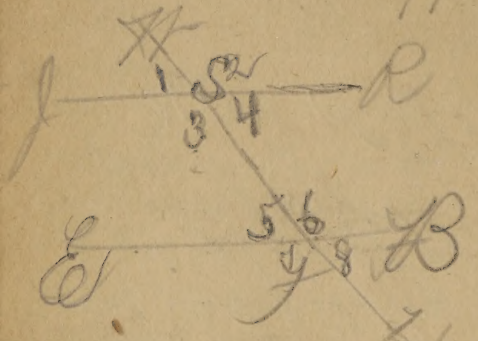
Prove. $\angle Z = \angle H$ & K.

1) $\angle Z = \angle S$ & Z (Th II Pt. 1)
2) $\angle H$ & $K = \angle S$ & Z (Th III)
3) $\angle Z = \angle H$ & K (Ax 8.)

Q. E. D.

Th 6

If two parallel lines are cut by a transv. the opposite int. ∠'s are equal



Prove ∠1 = ∠5
∠2 = ∠6
∠3 = ∠7
∠4 = ∠8

If ∠'s 1 & 5 of ∠'s 1 have same directions from their vertex S that ∠'s 5 & 7 have from their vertex F

2) ∠1 = ∠5 (Ax II)

Q.E.D.

Find the alternat-internal ∠'s are equal

Prove: ∠3 = ∠6 & ∠4 = ∠5.

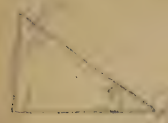
1) ∠3 = ∠2 (Th III)
2) ∠2 = ∠6 (Th I)
3) ∠3 = ∠6 (Ax 1)

Q.E.D.

Find the internal ∠'s on the same side are sup. of each other

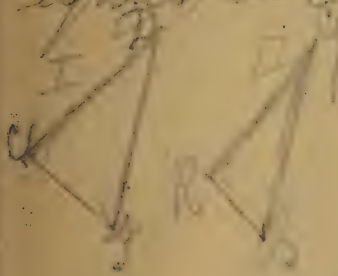
Prove: ∠6 = sup. of ∠4.

1) ∠3 = ∠6 (Th I)
2) ∠3 = sup of ∠4 (Th I Cor 1)
3) ∠6 = sup of ∠4 (Ax 2)



Case III.

If two triangles have two sides and the included angle of one respectively equal to two sides and the included angle of the other, the triangles are equal in all respects.

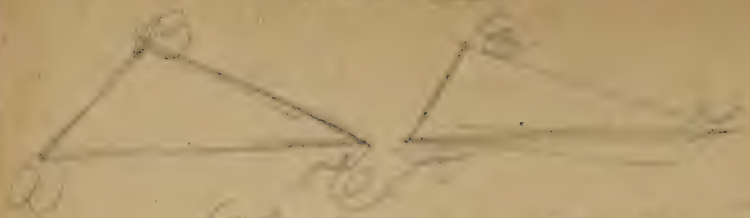


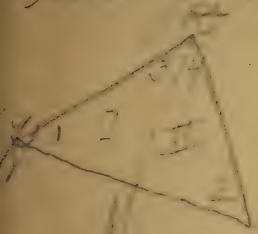
Hyp. In $\triangle$s I & II, $\overline{KA} = \overline{RD}$, $\overline{KB} = \overline{RE}$, and $\angle K$ = included $\angle$ of I = $\angle R$ = included $\angle$ of II.

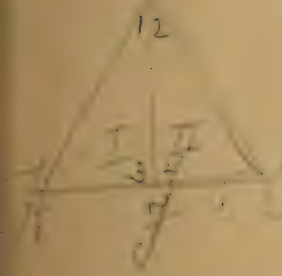
To prove $\triangle I = \triangle II$

Proof.

Pr. IX.

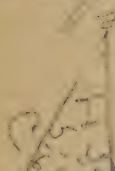






Th. 11

If two angles of a △ are equal the sides opposite the equal angles are equal and the △ is isosceles.



Hyp. ∠ Y = ∠ Z

Prove) EY = FY

△ EYF is isos.

Const. Draw BC to the base bisecting ∠ X

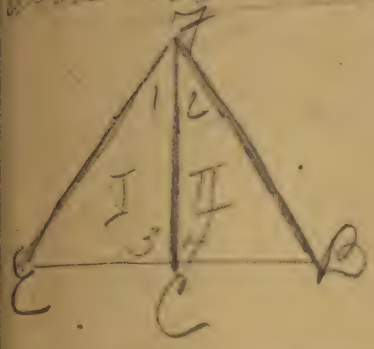
1) ∠ 1 = ∠ 2 (Hyp)
2) ∠ 3 = ∠ 4 (Const)
3) ∠ 5 = ∠ 6 (Cor. II Cor. 2)
4) CB = CB
5) △ 1 = △ II. (Th. II)
6) EB = FB (Sch. Th. III)

Q. E. D.

Note to Th. II

Theorem for 2

How can we prove a line from the vertex of the ∠ to the base of the △ are ≅

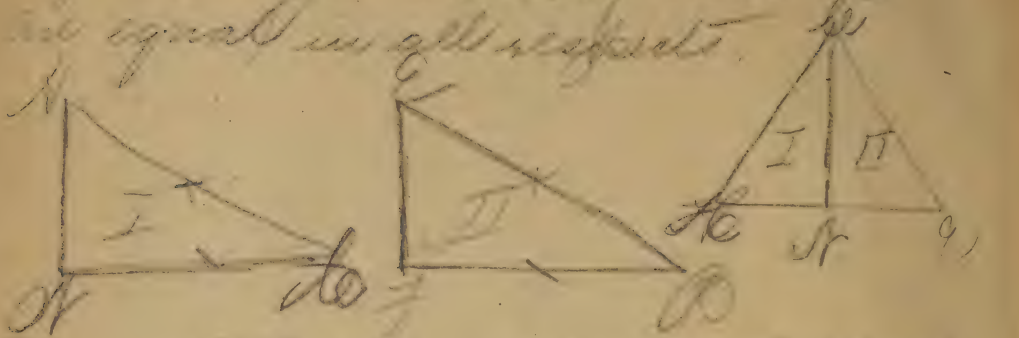


Prove: ∠1 = ∠2.
△I = △II
EC = CB

1) ∠E = ∠B
2) ∠3 = ∠4
3) ∠1 = ∠2 (Ins ∠s Cor ∠s)
4) △I = △II (Ins △s)
5) EC = CB (CPCTE)

No. XIV.

Two rt. △: have sum of hyp. & one leg = to sum of hyp. & other leg are equal in all respects.



Hyp. In rt. △ ABC & DEF, AB + AC = DE + DF, and BC = EF.

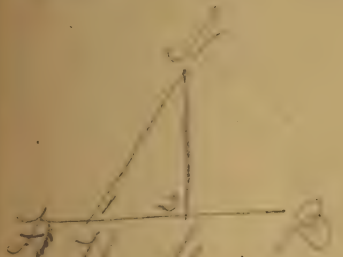
Prove △ I = △ II

Const. Place △ II so that EF coincides with BC, & so that △ II does not overlap △ I.

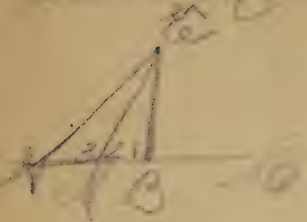
1) AB + AC = DE + DF.
2) AE = AB + BE (Ax. 2)
3) AE = DF (Ax. 1)

∴ △ I = △ II (Ax. 3)

L. Sr.

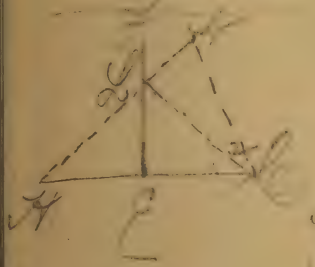

A
B

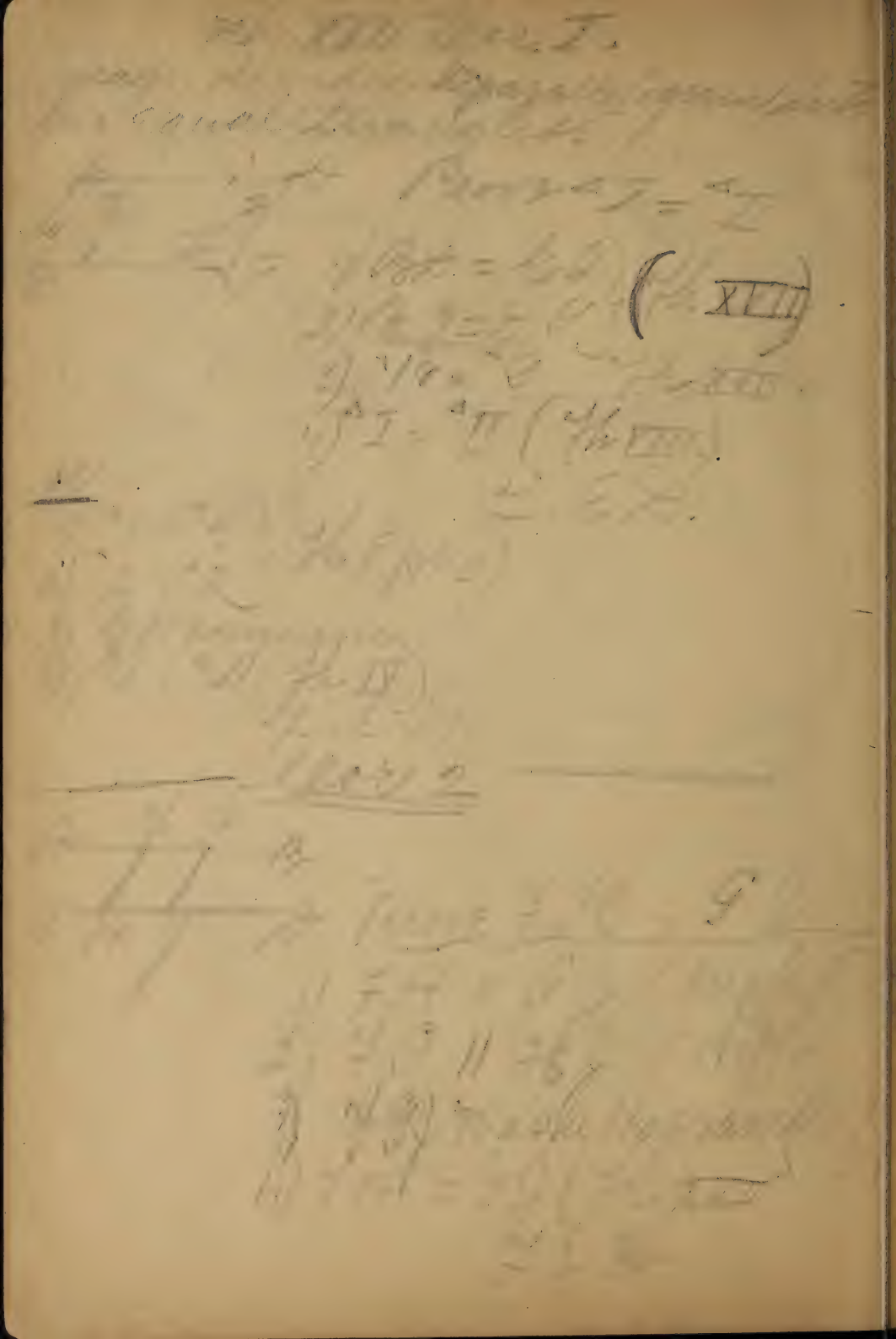
Gradpunkt.



1) $\angle$ = R. $\angle$ (= 90°) (= D)
2) $\angle$ = acute $\angle$ (for $\angle$ A III)
3) $\angle$ = obtuse $\angle$ (Def 4)
4) $\angle$ > $\angle$ (for $\angle$ Def III)
5) $\overline{CH} > \overline{CM}$ (for Def XII)

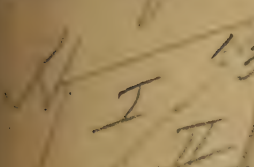
E. J.





Th. XIII

The opposite sides of a parallelogram are equal to each other



Given: Parallelogram

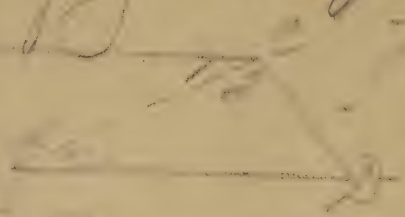
To prove:

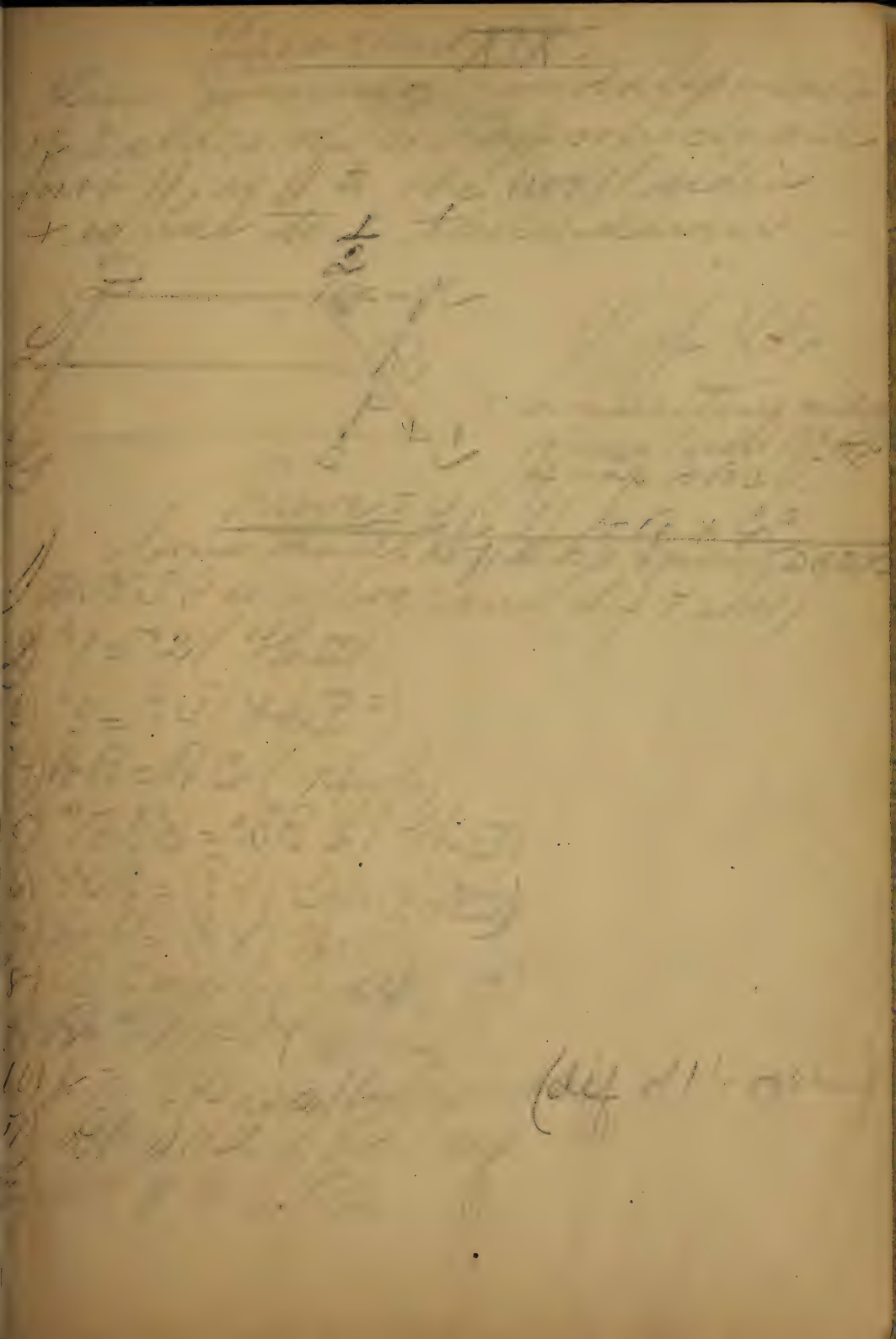
Proof:

1.
2.
3.
4.
5.
6.
7.
8.
9.

Th. XVIII.

The sides of a quadrangle are equal & the fig is a ||ogram.



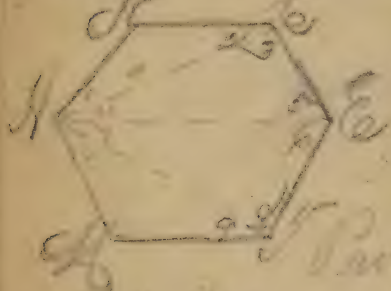


Th. XX

The sum of the interior angles of a polygon equal to $2nR$ taken as many times less two as the figure has sides.

Formula: $2R(n-2) =$ sum of int $\angle$'s.

$\frac{2R(n-2)}{n} =$ 1 ang.



$$\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$$

Prop. II. Inverse

$$\frac{a}{b} = \frac{c}{d}$$

Then $a - b : b = c - d : d$

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} - 1 = \frac{c}{d} - 1$$

$$\frac{a}{b} - b = \frac{c}{d} - d$$

$$a - b : b = c - d : d$$

Q. E. D.

Book I

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Lect. I

Ch. VIII.

Дано $a : b = c : d$ и $\frac{a}{b} = \frac{c}{d}$.

Доказать $a^n : b^n = c^n : d^n$.

1) $\frac{a}{b} = \frac{c}{d}$

2) $\frac{a^n}{b^n} = \frac{c^n}{d^n}$

3) $a^n : b^n = c^n : d^n$

Ч. т. д.

Дано $a : b = c : d = m : n$

Доказать $a + c + m : b + d + n = a : b$

$ab = a^2$

$cd = bc (= ab)$

$mn = bn (= ab)$

$a(b + d + n) = b(a + c + m)$

$a : a + c + m = b : (b + d + n)$

Ч. т. д.

Th I

$e : f = e : d$

$m : w = x : d$

Prove. $\frac{a}{m} : \frac{b}{w} = \frac{c}{x} : \frac{d}{y}$.

1) $ad = bc$ (Th I)

2) $mx = wc$ ( )

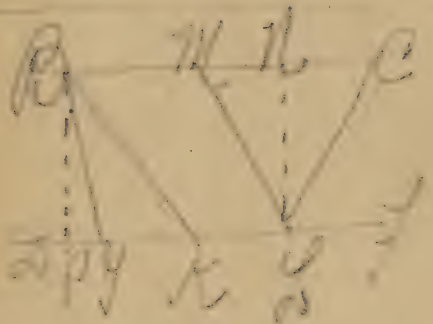
3) $adxy = bcwy$

4) $\frac{ad}{mwy} = \frac{bc}{mwy}$

5) $ayk : bwk = cxk : dwy$ Th II

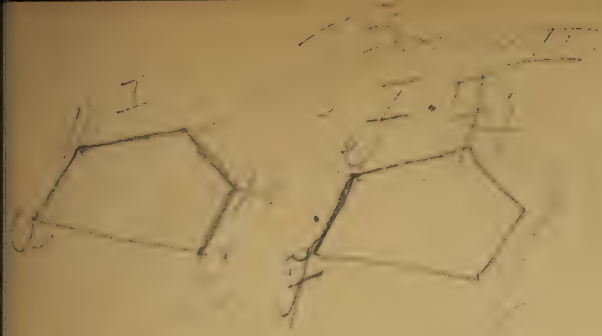
6) $\frac{a}{m} : \frac{b}{w} = \frac{c}{x} : \frac{d}{y}$.

Q. E. D.

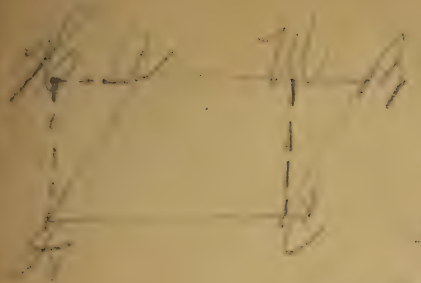


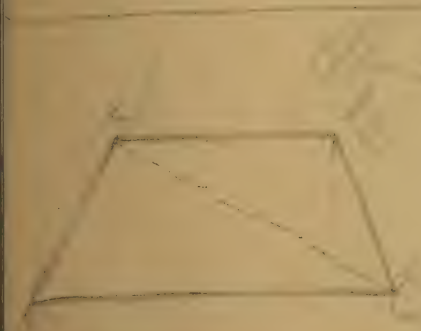
Ex. 1. Given
$\overline{AB} \cong \overline{CD}$ with
base $\overline{AC}$ & $\overline{BD}$
bisected by $\overline{EF}$.
Prove $\overline{EF} \perp \overline{AC}$ & $\overline{BD}$.

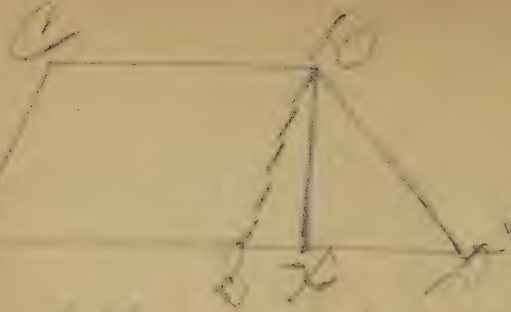
Proof. $\overline{BF} \cong \overline{FD}$ (Def. of bisect)
1.) $\overline{AB} \cong \overline{CD}$ Given
2.) $\overline{AF} \cong \overline{CF}$ Def. of bisect
3.) $\overline{BF} \cong \overline{DF}$ Def. of bisect
4.) $\overline{AC} \perp \overline{EF}$ & $\overline{BD}$ If a line bisects one of two $\cong$ segments & is $\perp$ to it, it is $\perp$ to the other.
5.) $\overline{EF} \perp \overline{AC}$ & $\overline{BD}$ Def. of $\perp$ lines
6.) $\overline{BF} \cong \overline{DF}$ (Def. of bisect)

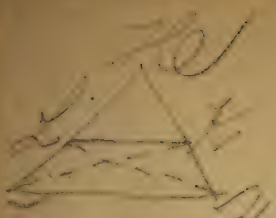


II









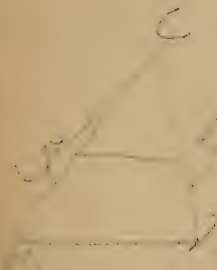
VI

Hyp. ABC Trapezoid
Line EF ∥ AB & CD

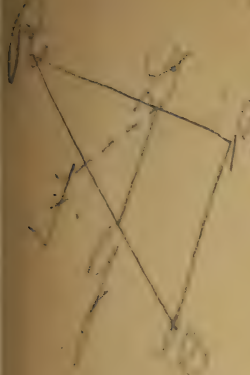
Prove $\frac{AE}{ED} = \frac{BF}{FC}$

Const. Draw AC & AF.

1) △ CDA & CAB have same altitude
2) $\frac{CDA}{CAB} = \frac{ED}{AB} = \frac{CE}{CA}$ (III. Th. 2)
3) CEF & CAB have same altitude
4) $\frac{CEF}{CAB} = \frac{EF}{AB} = \frac{CE}{CA}$ (III. Th. 2)
5) $\frac{ED}{AB} = \frac{EF}{AB}$ (II Ax. 1)
6) $\frac{AE}{ED} = \frac{AB}{EF}$ (II Th. 1)
7) $\frac{BF}{FC} = \frac{AB}{EF}$ (II Th. 1)
8) $\frac{AE}{ED} = \frac{BF}{FC}$ (II Ax. 1)



Th. III. Converse of Th. II



Given $\triangle ABC$ and P, Q, R in AB, BC, CA respectively such that $AR : RC = BQ : QC = AP : PB$

To prove $PQ \parallel AC$, $QR \parallel AB$, $RP \parallel BC$

Suppose PQ is not $\parallel$ to AC. Then through P draw PQ' $\parallel$ to AC meeting BC in Q'.

Then $PQ' \parallel AC$

$\therefore BP : PA = BQ' : Q'C$ (Th. II)

But $BP : PA = BQ : QC$ (Given)

$\therefore BQ' : Q'C = BQ : QC$

$\therefore BQ' + Q'C : Q'C = BQ + QC : QC$

$\therefore BC : Q'C = BC : QC$

$\therefore Q'C = QC$

Similarly $AR = RC$

$\therefore$ Each equals $\frac{1}{2}AC$

$\therefore Q'$ coincides with Q

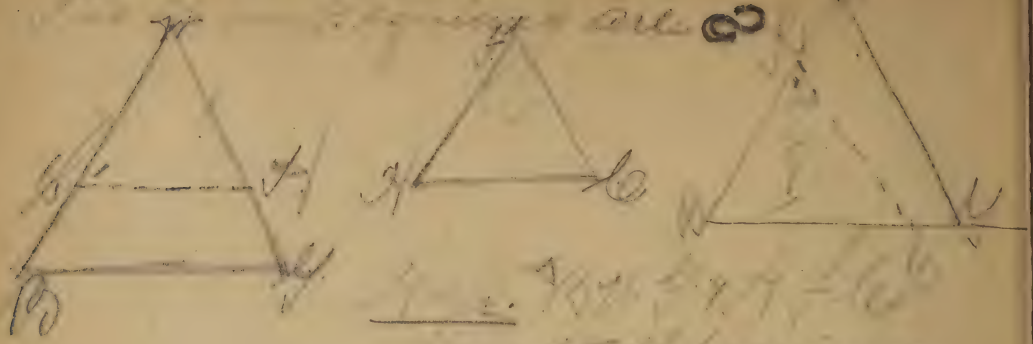
$\therefore PQ \parallel AC$

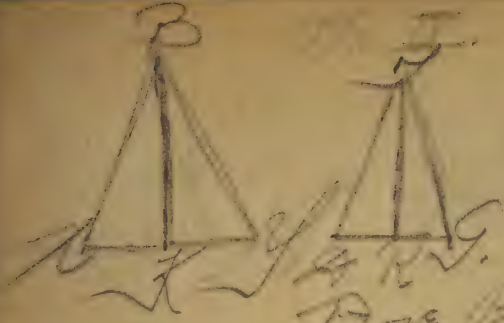
Similarly $QR \parallel AB$

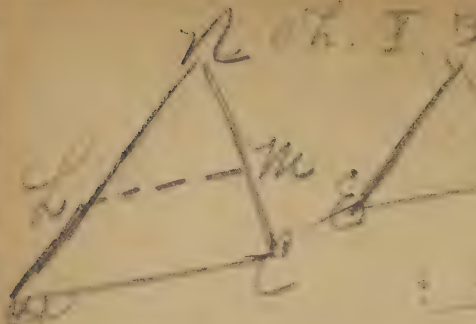
$\therefore$ Each angle of $\triangle PQR$ is equal to the corresponding angle of $\triangle ABC$

$\therefore \triangle PQR \sim \triangle ABC$

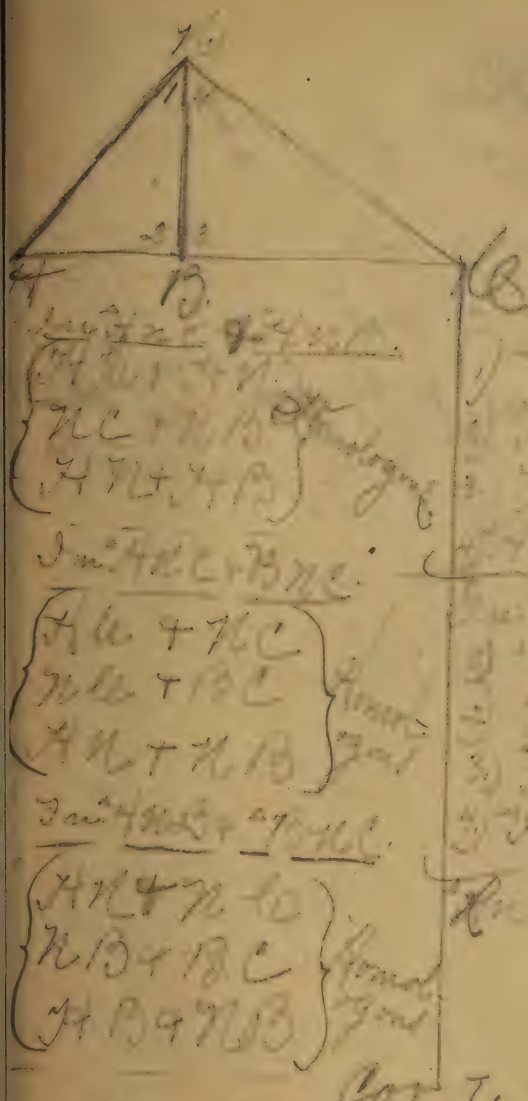
$\therefore$ Each angle of $\triangle PQR$ is $< 60°




B
X
Y
R
G

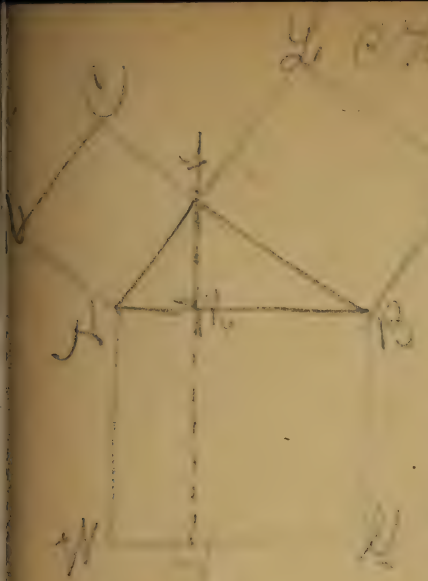


№ II.



Ejercicios

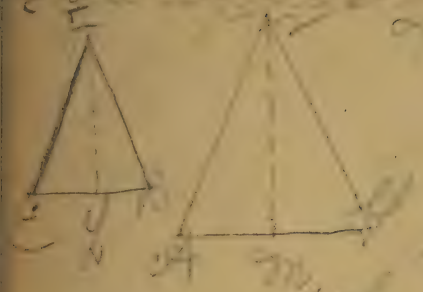
XII



Дано: $\angle KAB$

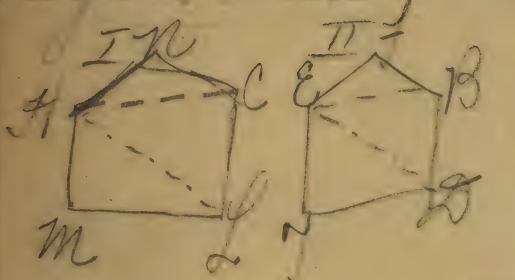
Дано: $\angle KAB$

Дано: $\angle KAB$



XII

No. XV



Hyp. ∞poly. I ~ II

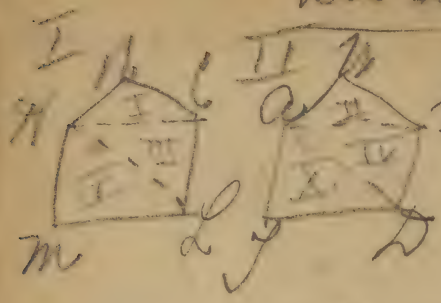
Prove I of I : P of II =
HN : EF

Cons. Draw diags

1) HN : EF = NC : FB = CL : BD = LM : DF
2)
3) HN + NC + CL + LM + MH : EF + FB + BD + DF + FE = HN : EF
4) HN + NC + CL + LM + MH = P of I.
5) EF + FB + BD + DF + FE = P of II.
6) P of I : P of II = HN : EF (subst in 3)

L. E N.

Th XIV Pt 2



Hyp ∞ Polys I, II

Prove I : II = MN² : AW²

Cons. Draw diag AC, AE, EC

from A & G & D to C.

1) Polys are divided into same no. of Δs (by XII)

2) ΔI : ΔII = AC² : MN² (by XIII)

3) ΔIII : ΔIV = AC² : MN² (by XIII)

4) ΔI : ΔII = ΔIII : ΔIV (by 2, 3)

5) ΔIII : ΔIV = AE² : MN² (by XIII)

6) ΔV : ΔX = AE² : MN² (by XIII)

7) ΔII : ΔI = ΔV : ΔX (by 5, 6)

8) ΔIII : ΔII = ΔV : ΔX (by 4, 7)

9) ΔI + ΔIII + ΔV : ΔII + ΔIV + ΔX = ΔI : ΔII (by 8)

10) I : II = AC² : MN² (by 9 & XII)

11) I + III + V = P I. (by def.)

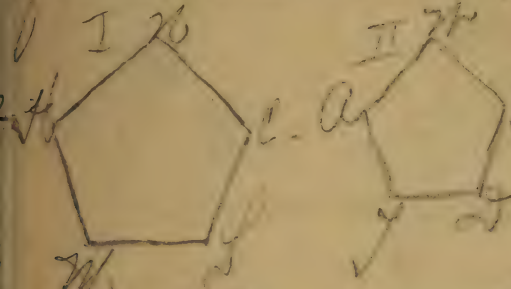
12) II + IV + X = P II. (by def.)

13) P I : P II = AC² : MN² (subs. in 9)

Q. E. D.

Th. XVII.

Similar polygons of the same number of sides are ∽



Hyp. The key for ∽
I + II

Conc. I ∽ II.

1) Sum of ∠s = (n − 2) 2 rt ∠s (I. Cor. XV)
2) ∠A = ∠A'; ∠B = ∠B'; ∠C = ∠C'; etc. (Hyp.)
3) I + II mutually equiangular
4) AB : A'B' = BC : B'C' = CD : C'D' = etc. (I. Th. IV)
5) AB : A'B' = BC : B'C' = CD : C'D' = etc.
6) AB : A'B' = BC : B'C' = CD : C'D' = etc.
7) I ∽ II (∽ ∠s & ∽ sides are homologous)
∴ I ∽ II (Def. of ∽ polygons)

Q. E. D.

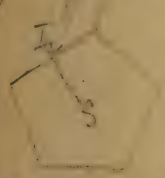
Ch. XIII

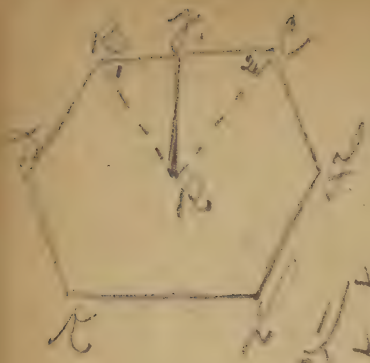
Prove 1) $\angle A' = \angle A$

Cors.

I. [illegible]

II. [illegible]





1) ∠ KLM = ∠ BRC = ∠ HSM (∠ с сторонами ⊥ ед.)
2) ∠ 2 " " = ∠ 4 " "
3) ∠ R " " = ∠ S
4) △ BRC ∽ △ HSM (2 ∠а I. ∠ III вид)
5) LC : SH = BC : HM (II : Th IX)

Ч. Т. Е. Д.

1) PQ : GS = BC : HM (II Th XIII Cor 3)
2) Perim of I : Perim of II = BC : HM (I Th XV)
3) " " " I : " " III = BC : HM (Ax. 11)

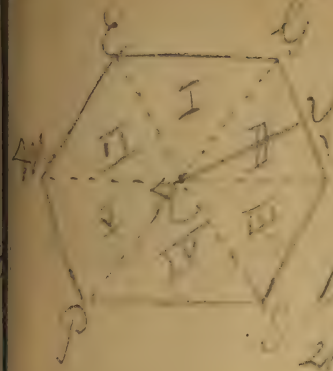
Ч. Т. Д.

1) I : II = BC² : HM² (II Th XI)
2) KL : SG = BC : HM (II Th VII Cor 1)
3) KL² : SH² = BC² : HM² (Ax 5.)
4) I : II = KL² : SH²

Ч. Т. Д.

№ XVIII

Фиг. Разбор



Сечение диаг. $I = \frac{1}{2} P \times n y$.

1 диаг. $I = \frac{1}{2} n y \times E \cdot D$

2 " " $II = \frac{1}{2} n y \times$

3 " $III = \frac{1}{2} n y \times$

4 " $IV = \frac{1}{2} n y \times$

5 " $V = \frac{1}{2} n y \times$

6 " $VI = \frac{1}{2} n y \times$

$\left.\begin{matrix} \\ \\ \\ \\ \\ \end{matrix}\right\} \times \frac{1}{2} n y$

7 $P_{n y} = \frac{1}{2} n y$

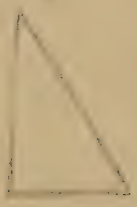
8) Работа

$= \frac{1}{2} n y \times$ (размер сечения)

Е. Д

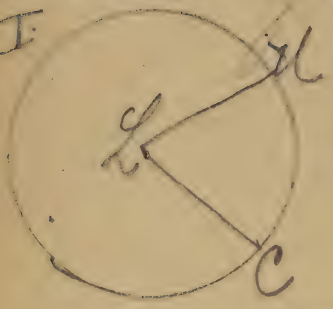
Practical Questions.

1) Perim = 25' + 20' + 36' + 30'
   Area = 25' × 30' (Π Thm □) 270.
2) area = 30 × 12. (Ⅲ Thm. Ⅳ) 360 sq ft.
3) " = $\frac{1}{2}$(14 × 8.) (Ⅱ Thm Ⅸ)
4) Trap. area = $\frac{1}{2}$(alt + sum of ∥ sides)
5) 3 – 4 – 5.
6) 30 ft.
7)

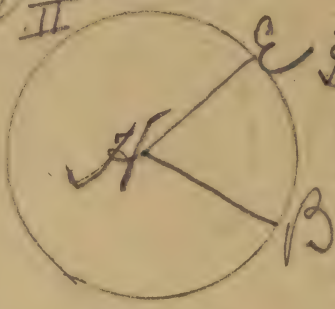


circle or
angles at the center
equal

I.



II.



Hyp Equal circles I & II. with ∠ = ∠ at the centers ∠ & H.

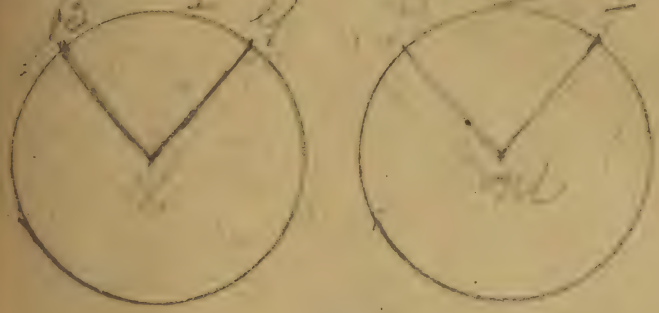
Prove. $\overset{\frown}{MC} = \overset{\frown}{EB}$

By superposition Place ⊙ I on ⊙ II so that pt. L will fall on pt. H and side LM will fall upon HE and the pt M will fall upon pt E (Cor def Rad) Then as ∠ = ∠ HB & ∠C will take the direction of HB & coincide with it and pt. C will fall upon pt B (or as) and as MC & EB represent any pt in the circ. of the ⊙ $\overset{\frown}{MC}$ will coincide with $\overset{\frown}{EB}$ (Number I & II would not be perfect ⊙.

$\overset{\frown}{MC} = \overset{\frown}{EB}$ Q. E. D.

Th. I Pr. 2.

In the same ⊙ or equal ⊙ equal arcs are subtended by equal angles at the center.

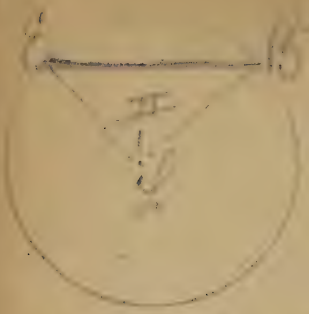


Hyp. In ⊙ I ...

Ratio ... =

... equal ... angles ... at center

$$\text{Ratio } \frac{\text{Area } \odot \text{ I}}{\text{Area } \odot \text{ II}} = \frac{\text{Area}}{\text{Area}}$$


E
B
H
G

H
a
M

Conf.

B
A
1
2
3
4
G

Th. I Cor. 1.

In the same ⊙s or equal ⊙s equal segments are bounded by equal chords & equal arcs; & conversely in the same ⊙ or equal ⊙s if segments are bounded by equal arcs & equal chords the segments themselves are equal.

Hyp

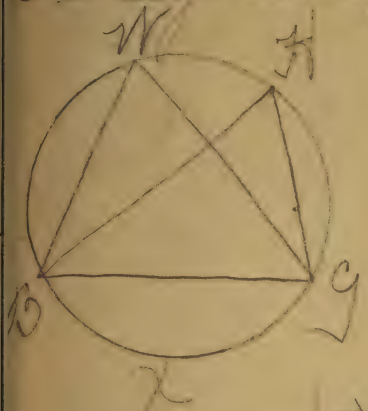
I.

⊙s ABC & EFG ⊃ I & II with equal segments

II.

Th. I Cor 2

All angles inscribed in the same seg = seg. are equal



Hyp. ⊙ BZGW with ∠BWG = ∠BHG inscribed in the seg BZWHG.

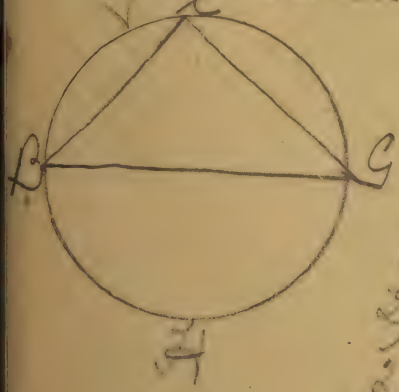
Prove ∠W = ∠H.

---

1) ∠BHG is measured by ½ BZG (III Th)
2) ∠BWG " " " "
3) ∠BHG = ∠BWG (Ax 6)

Th. I Cor 3

Every ∠ inscribed in ○ = 1 R ∠.



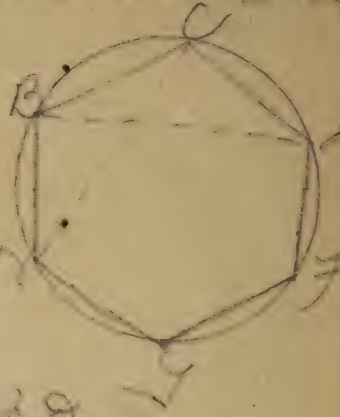
Hyp. ○ BZG with ∠BZG

Prove ∠BZG = 1 R ∠

1) BHG measured = ○ (Ax 2)
2) ∠BZG is measured by ½ BHG (III Th I)
3) ∠BZG " " ½ Cir of the ○
4) ∠BZG " ¼ ○ (IV Th II Cor)

Th VI

Each equilateral polygon inscribed in a ⊙ is regular.



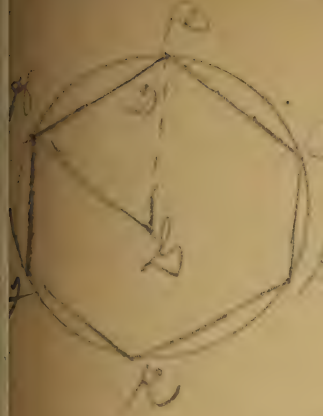
Hyp ⊙ Equilat △ABCDEF inscr in ⊙ BCDEF.

Prove △ABCDEF is reg.

Const. Draw chs. AC + BD.

1) $\overline{AB} = \overline{BC} = \overline{CD}$ (Hyp.)
2) $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD}$ (III Th II)
3) $\overset{\frown}{AB} + \overset{\frown}{BC} = \overset{\frown}{BC} + \overset{\frown}{CD}$ (Ax 1.)
4) $\overline{AB} + \overline{BC} = \overline{ABC}$ (Ax 7)
5) $\overline{BC} + \overline{CD} = \overline{BCD}$ (Ax 7.)
6) $\overset{\frown}{ABC} = \overset{\frown}{BCD}$ (Ax 6)
7) $\overline{AC} = \overline{BD}$ (III Th 4.)
8) $m\angle ABC = m\angle BCD$ (III Th 5. Cor 1)
9) $\angle B = \angle C$ (III Th 5 Cor 2)
10) In same way $\angle A = \angle B = \angle C = \angle D = \angle E = \angle F$
11) ∴ △ABCDEF is equilat. & equiang. Q.E.D.

XI



Κανονικό εξάγωνο ΑΒΓΔΕΖ εγγεγραμμένο σε κύκλο (Ο, R). Να αποδειχθεί ότι η πλευρά του εξαγώνου είναι ίση με την ακτίνα του κύκλου.

Δοθέν: Κύκλος (Ο, R), εξάγωνο ΑΒΓΔΕΖ εγγεγραμμένο στον κύκλο.

Ζητούμενο: Να αποδειχθεί ότι ΑΒ = R.

Απόδειξη:

Οι πλευρές του εγγεγραμμένου κανονικού εξαγώνου είναι χορδές (ορισμός κανονικού πολυγώνου).

Άρα ΑΒ = ΒΓ = ΓΔ = ΔΕ = ΕΖ = ΖΑ = χορδή του κύκλου.

ΑΒ = ΒΓ = ΓΔ = ΔΕ = ΕΖ = ΖΑ (III κύκλος)

Ο κύκλος χωρίζεται σε 6 ίσα τόξα.

Κάθε τόξο = 1/6 του κύκλου = 60°

Άρα ∠1 = 60° (III κύκλος)

Η ∠1 = ∠ΑΟΒ (κεντρική)

Η ΟΑΒ = ισόπλευρο

1 + 2 + 3 = 2 ∠ορθές

1 + 2 = 180°

1 = 2 (I ΙΣΟΤΗΤΑ)

Άρα 1 = 60°

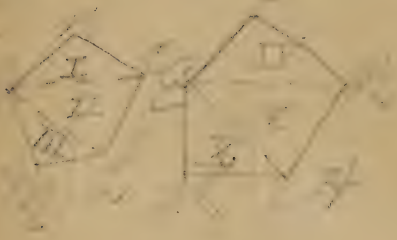
Άρα 2 = 60°

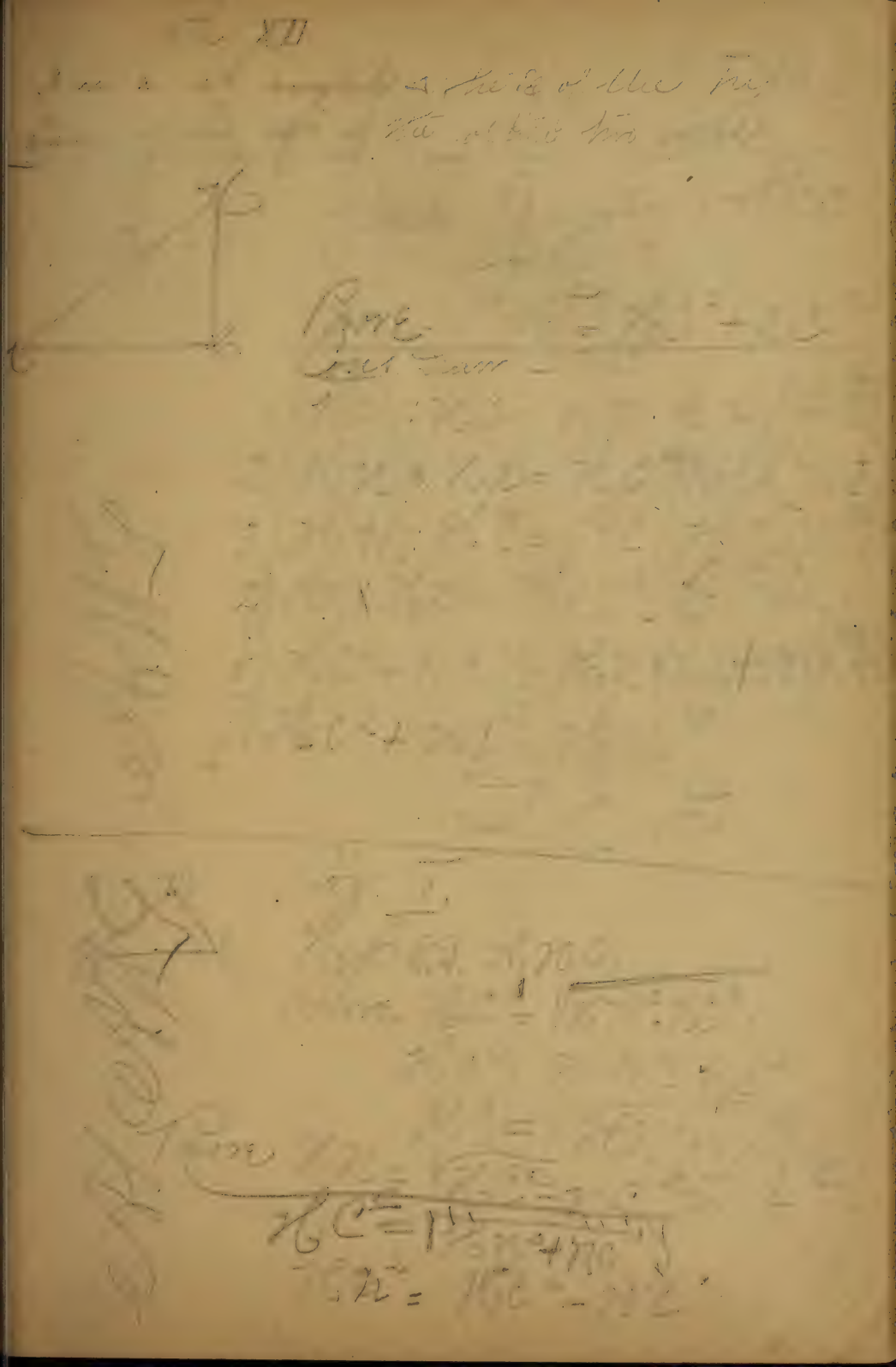
Η ΑΒ γειτονική

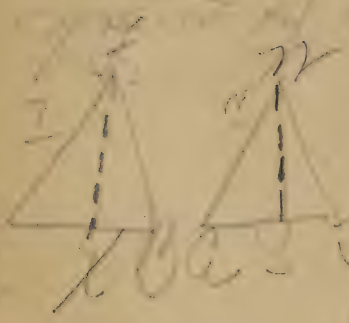
Άρα

Η 3 = Η ΑΒΓΔ = ...

Prove area of Circle.


I
II
III
IV
V





Theorem: Every reg. ⬠ is =

Hyp. Given reg. pentagon

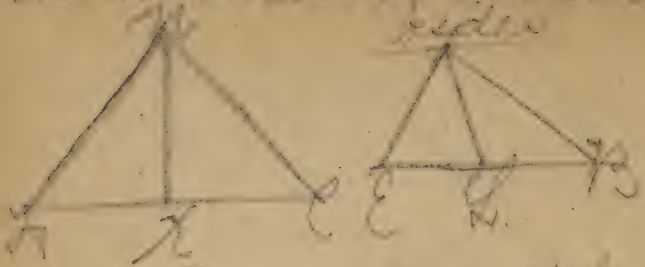
Prove,

1) $\angle C = \angle B$.
2) $\overline{AC} = \overline{AB}$
3) $CE = FB$
4) $\triangle I = \triangle II$ (I Ax. VIII)
5) $AE = AB$ (Sub. I Ax. VIII.)

Prove

1) AS ; $\overline{AE}$; $\overline{BC}$; $\angle$ (II, I.)
2) $\overline{AC}$; $\angle$; $\overline{AE}$ = $\overline{AC}$; $\angle$; $\overline{AB}$ (I, 1)
3) $\overline{AC}$; $\overline{AS}$; $\angle$; $\angle$; $\overline{AB}$ = $\overline{AC}$; $\angle$; $\overline{AE}$ (I, 2)
4) $\angle$ + $\angle$ = $\angle$ (I Ax. IV.
5) $\angle$ + $\overline{AE}$ = $\overline{AB}$
$\angle$ = $\angle$

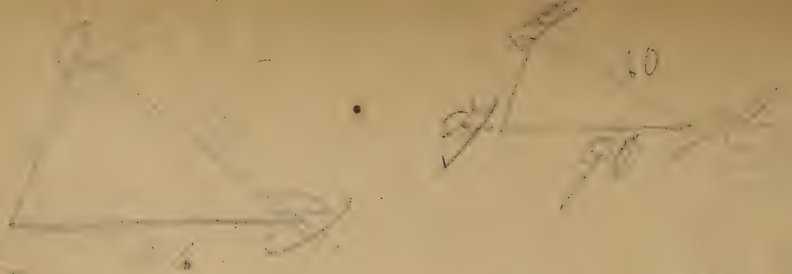
The medians of $\triangle$s are proportional to their sides

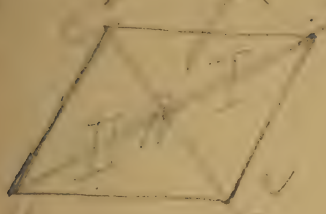


Hyp. $\triangle ABC \sim \triangle EFD$
medians MX & FZ

Prove $MX : FZ = AC : EB$.

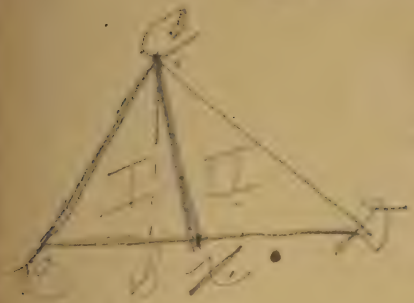
Proof





Prove $\vec{II} = -\vec{II}$; $\vec{III} = -\vec{I}$

To what pt in the base of Δ a given line be drawn from vertex to bet. the Δ into 2 = parts



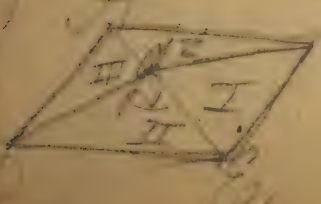
Bisect the base. Then Δ I + II will be = & have equal bases & altitudes according to Th. II. I = II

Prove that a line joining vertex of Δ to any pt. of base in same ∥ to line base = Δ's.

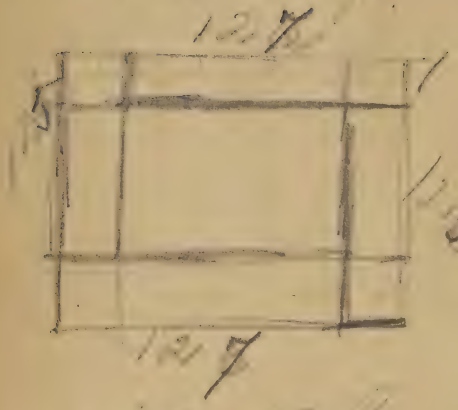
Cons. fig. Extend base to B. Then Δ ABC & DBC have equal altitudes & equal bases

---

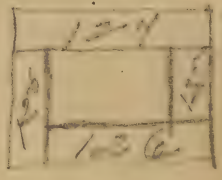
Prove a line to bisect a Δ from any pt in side of Δ must be divided into 2 parts proportional

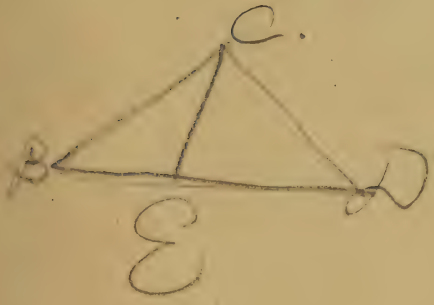


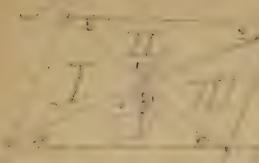
An eq. plot of ground with a path 4 ft wide
on the inside of the fence
what will be expense of gravelling
the path at 12 cts per sq. yd.



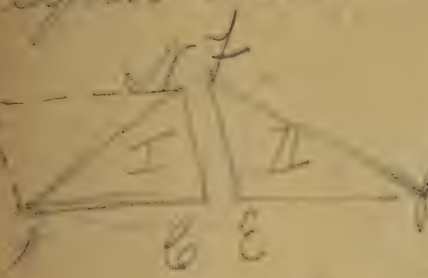
127
1 2
254
250
504
60.48




C
B
D
E


II
I
III

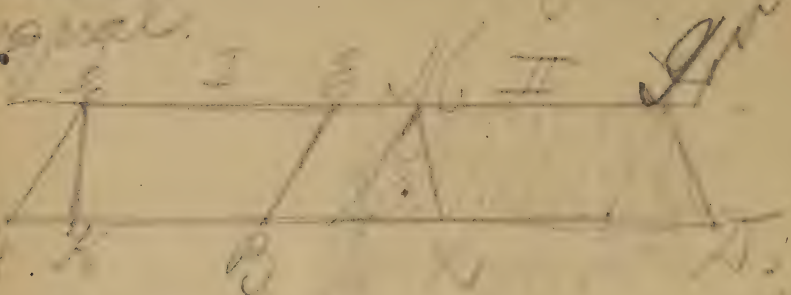
2 △ having 2 adjacent sides & incl. angle equal each to each & the angles opposite homologous sides equivalent



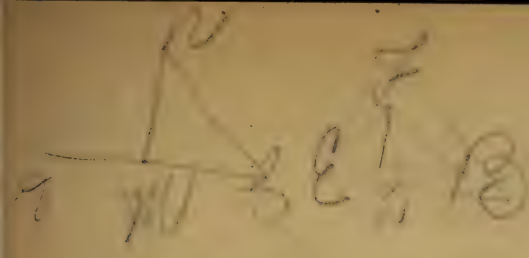
Sent. I = II.

~~Cor. Therefore~~ ~~the~~ ~~△~~ ~~ABm~~
~~are~~ ~~by~~ ~~Prop~~

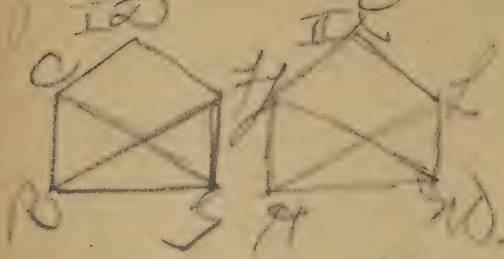
1) AM = MB (Def. Mid. Pt.)
2) I = II (Hyp.)
3) ~~I = II~~ (Const.)



Prove ∥ I = ∥ II.



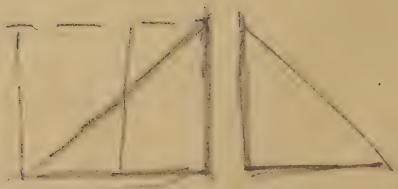
Perim of regl Pgn of the same no of sides are to each other as their sides



Prove Perim of I : Perim of II = BF : HL

BC : HJ = CD : JK = DF : KL = FG : LM = GB : MH

K
B
F
A
E

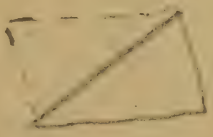


A unit of measure is 1 of the equal parts into which a quantity may be divided.

The unit of measure of a line is one of the equal parts in which a line may be divided. This is called a linear unit.

The unit of meas of a surface is the sq measure surface described on a linear unit. This is called the square unit.

A straight line drawn from the vertex of a Δ bisecting the base is called the 2nd. & the portion of it intercepted between the vertex & the base $= \frac{1}{2}$ the base



Ex. a : b = c : d

1) am : bm = cm : dm

2) $\frac{a}{m} : \frac{b}{m} = \frac{c}{m} : \frac{d}{m}$

a : c = b : d (Ex. III)

2) $\frac{a}{c} = \frac{b}{d}$

3) $\frac{am}{cm} = \frac{n}{n}$

4) $\frac{a}{c} \times \frac{m}{m} = \frac{b}{d} \times \frac{n}{n}$

5) $\frac{am}{cm} = \frac{bn}{dn}$

6) am : cm = bn : dn

1) a : b = c : d

2) mn : n = cn : n

3) am : bn = cm : dn (Ex 5) Ex 2

4) $\frac{a}{m} : \frac{b}{n} = \frac{c}{m} : \frac{d}{n}$ Ex 4

Def Page 91.

If 3 or more ratios are proportional the 1st of the 1st is to the 2nd as the 1st of the last is to the sum of the 2nds.

Hyp. $a : b = p : q$
$b : c = q : r$

Prove $a : \frac{b}{2} = p : \frac{q}{2}$

Original

1) $a : b = p : q$ (Hyp.)
2) $b : c = q : r$ (Hyp.)
3) $a : p = b : q$ (Th. II)
4) $c : c = b : q$
5) $a : p = c : r$
6) $a : c = p : r$ (Th. III)

7) $a : b = r : q$ (Th. II)
8) $a : c = b : r$ (Th. II)

Q. E. D.

$a : b = c : d$

$p : n = r : s$

Pour $\begin{cases} a \times p : b \times n = c \times r : d \times s. \\ \frac{a}{b} : \frac{r}{n} = \frac{c}{r} : \frac{d}{s} \end{cases}$

1) $\frac{a}{b} = \frac{c}{d}$

2) $\frac{p}{n} : \frac{r}{s}$ Le tout

3) $\frac{a \times N}{b \times n} = \frac{c \times r}{d \times s}$

4) $a \times p : b \times n = c \times r : d \times s$ CQFD

---

1) $a \times d = b \times c$ Le tout

2) $b \times s = n \times r$

3) $(a \times p) \times (d \times s) = (b \times n) \times (c \times r)$

4) $a \times p : b \times n = c \times r : d \times s$ CQFD

---

1) $\frac{a \times d}{b \times s} = \frac{b \times c}{n \times r}$

2) $\frac{a}{p} : \frac{d}{s} = \frac{b}{n} : \frac{c}{r}$

3) $\frac{a}{p} : \frac{b}{n} = \frac{c}{r} : \frac{d}{s}$

4) $\frac{a}{p} : \frac{d}{s} = \frac{b}{n} : \frac{c}{r}$

$a : b = a + c + e : b + d + f$

1) $a : b = c : d$

$a + b : a - b = c + d : c - d$

2) $a + b : b = c + d : d$ V

3) $a - b : b = c - d : d$ VI

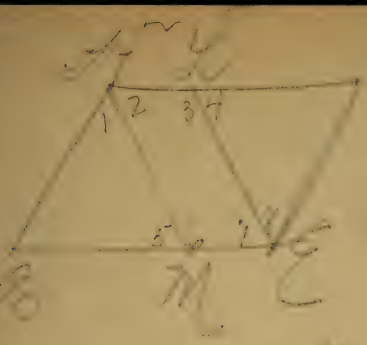
4) $a + b : c + d = b : d$ III

5) $a - b : c - d = b : d$ III

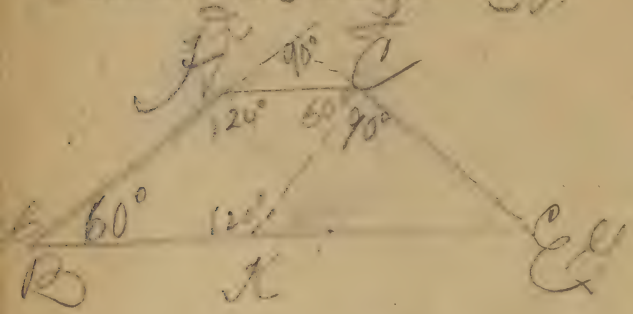
6) $a + b : c + d = a - b : c - d$ III

6) $a + b : a - b = c + d : c - d$ III

Δ. Ed.

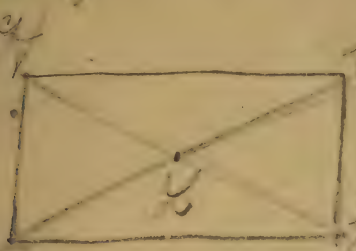


The legs of a trap are ⊥ to each
other & the oblique angle ∠ = 60°
and oblique ∠.



1) ∠a = comp of ∠ 60°.
2) ∠x = supp 60° = 120°
3) ∠F = 120° ∠ Ext ∠
4) ∠b = ∠e (Alt ∠s)
5) ∠y = 30°

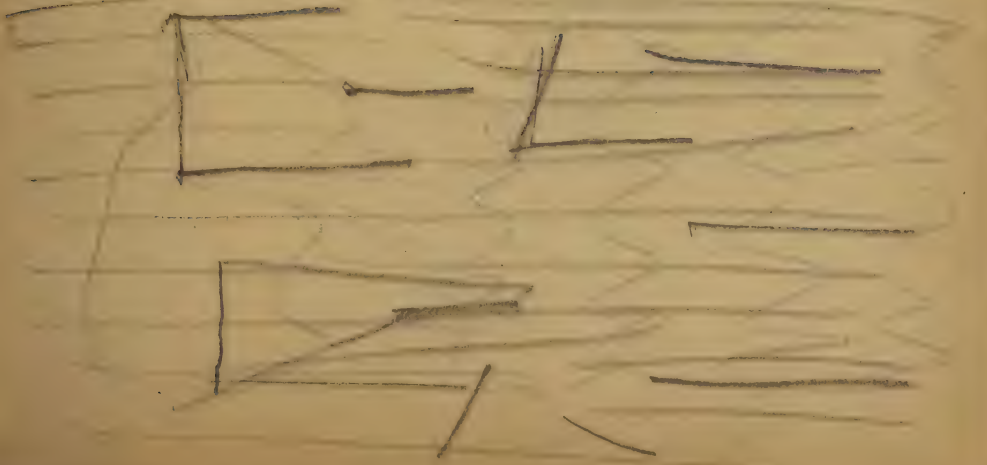
The diagonals of a rectangle are equal.



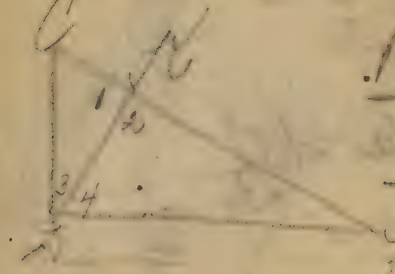
Prove $AE = EB$

1.
2.
3.

180
90
2



If from the vertex of an ∠ a ⊥ be supposed upon the hypotenuse the ∆'s formed will be equiangular to the orig ∆ & to each other



Prop II ∆'s & ∠'s of ∆'s equal

II ∆ ABC & DEF ∠ A = ∠ D ∠ B = ∠ E

III ∠ C = ∠ F ∆ ABC ~ ∆ DEF

II. 1) ∠ 1 = ∠ A
2) ∠ 2 = ∠ B
3) ∠ 3 = ∠ C (Cor 6 th)

∠ 2 = ∠ E
∠ 3 = ∠ F
∠ 4 = ∠ C

∠ 1 = ∠ D
∠ 4 = ∠ E
∠ 3 = ∠ F

$(c+d)a = (a+b)c$

$a \times c + a \times d = a \times c + b \times c$

$a \times d = b \times c$ (ax 2.

$a : b = c : d$ (Th. II) Theorem.

$m : n = a^{2-1} : 2$

$m : n = (a+b) : (a-b)$

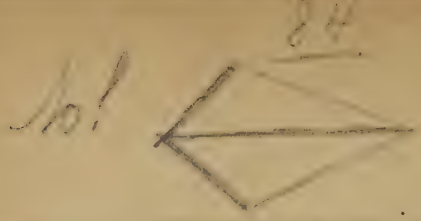
$m : a+b = a-b : n$ Th II

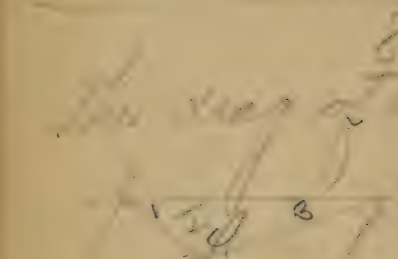
A mean proportional between
2 quantities is equal to the
square root of their product.

a) $a : x = x : c$

Prove $x = \sqrt{a \times c}$.

1) $a : S = x : c$ (typ.)
2) $a \times c = S \times S$ (Th. I)
3) $S^2 = a \times c$ (Th. I)
4) $S = \sqrt{a \times c}$ (ax 5.)
5) Q. to. D.

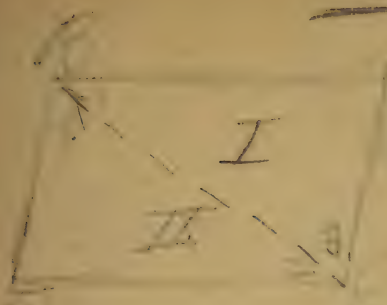


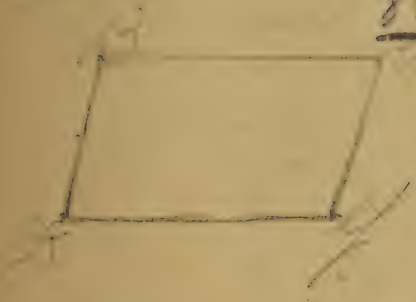


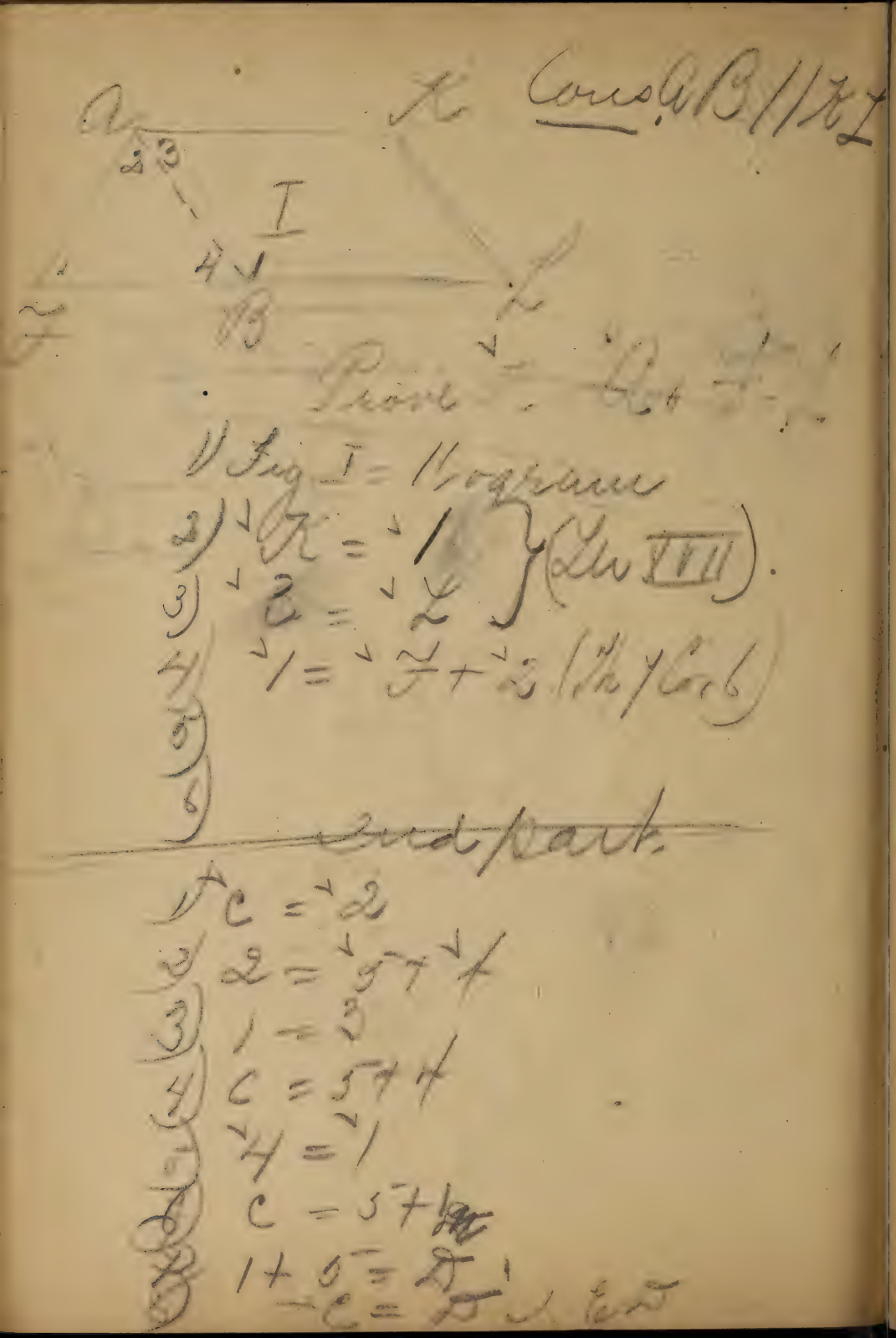
Теорема. $\angle A = \angle C$

1) $\angle 1 = \angle 2$ (как $\perp$ к $\parallel$)
2) $\angle 3 = \angle 4$
3) $\triangle I = \triangle II$ (по I)

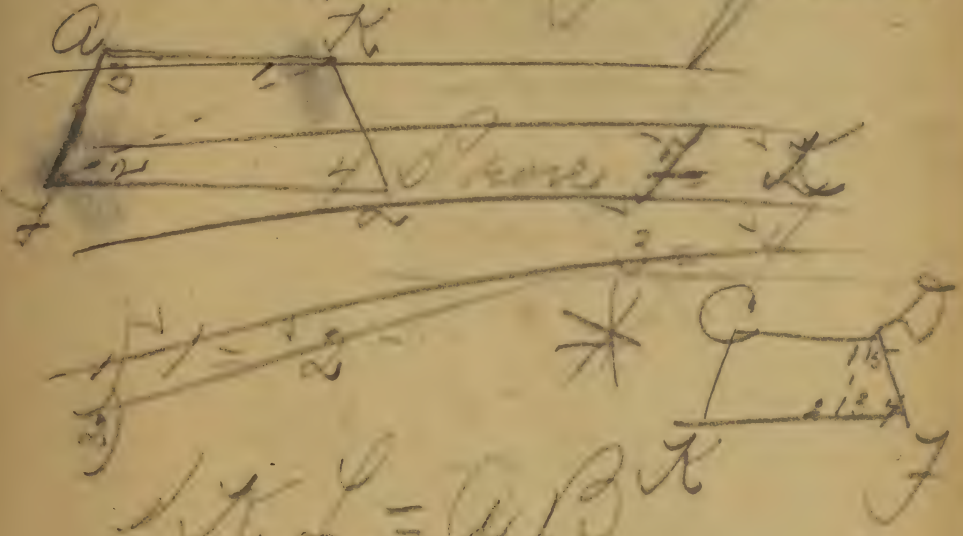
$BH = \perp_{\text{к}} AK$
$CF = \perp_{\text{к}} CE$
$\angle 1 = \angle 2$
$\triangle I = \triangle II$ (по III)


I
II



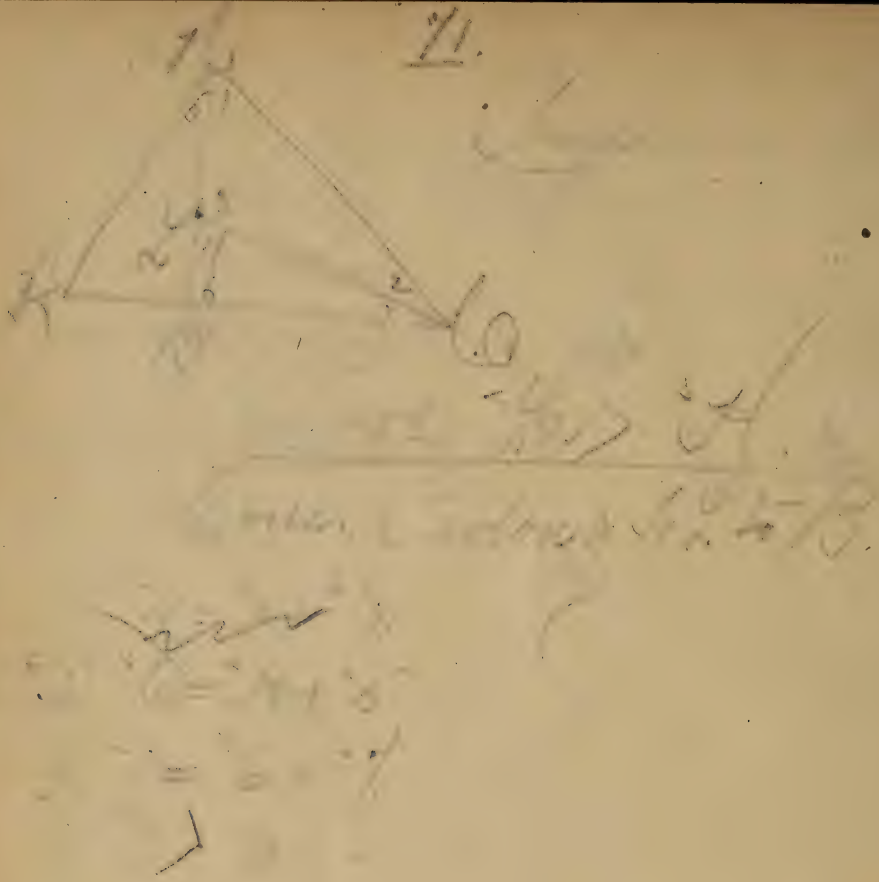

Cons. AB // KL
I
Prove
Fig I = Hexagon
(Def VII).
End Part.

If two ∠s have their sides ∥ each to each and = they are ∥ and have their sides also ∥ each to each.

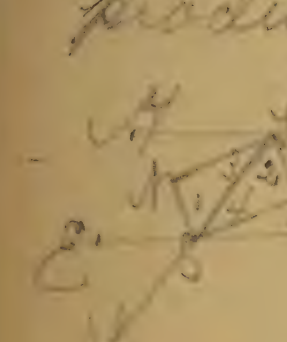


1) ∠K = ∠B
2) ∠Z = ∠C
3) ∠B = ∠Z.
4) AB ∥ ZF.
5) ∠A = ∠Z
6) ∠K = ∠F (alt ∠s)
7) ∠Z = ∠L.

∴ ∠L = ∠F.



If the interior angles formed by a secant crossing 2 par. lines on the same side of the secant are produced until they meet



a parallelogram

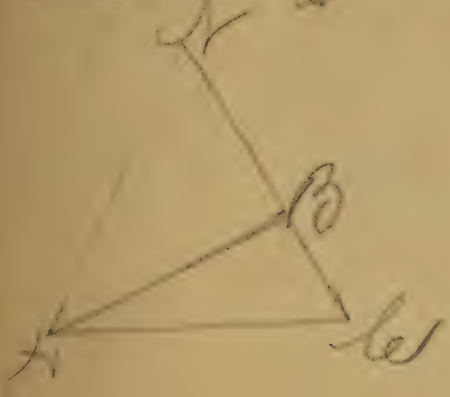
Proof: Let ∠'s A & B be interior ∠'s formed by secant ℓ crossing ∥ lines $\overleftrightarrow{AM}$ & $\overleftrightarrow{BK}$ on the same side of ℓ.

1) ∠'s A & B are supplementary (Th. V)
2) $\frac{6}{2} = \frac{6}{2}$ (Ax. 7)
3) $\frac{2}{2} = \frac{1}{2}$ (Cancel)
4) $\overleftrightarrow{AM} \parallel \overleftrightarrow{BK}$ (Th. VI)
5) $\overleftrightarrow{AB} \parallel \overleftrightarrow{AM}$, $\overleftrightarrow{BK}$ (Th. II)

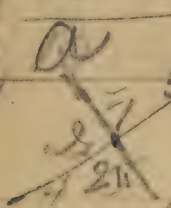
In same way

6) ∠'s A & B are supplementary
7) ∠'s A & B + ∠'s A & B = 2 rt ∠'s
8) 1 = 1
9) ∠'s A & B = 1 rt ∠

If from either end of the base of an isos. Δ a line be drawn perpendicular to the opposite side, the angle between that & the base will be $\frac{1}{3}$ the last angle of Δ



A pair of lines parallel to the bisected and this line joining the vertices through the centres of circles.

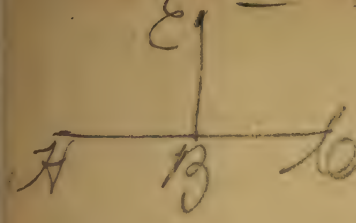


Join A, B. Let AB meet the circles in E, F.

Let the tangents at E, F meet in G.

Prove. AGB = a right angle.

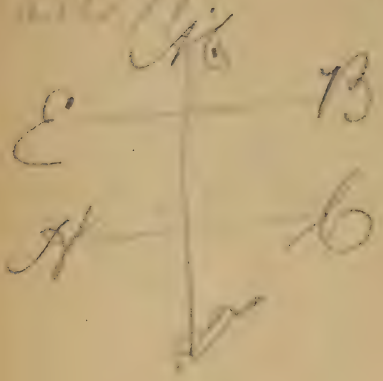
1) ∠1 = ∠2 (Tan Th)
2) ∠3 = ∠4 (Tan Th)
3) ∠G = ∠A
4) ∠1 = ∠II (Ext ∠ Th)
5) ∠A + ∠B = 2 rt ∠s (Int ∠s Th)
∴ ∠G = a rt ∠.

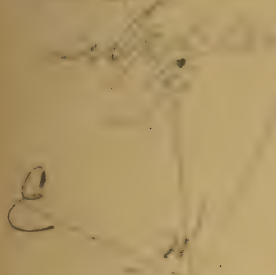


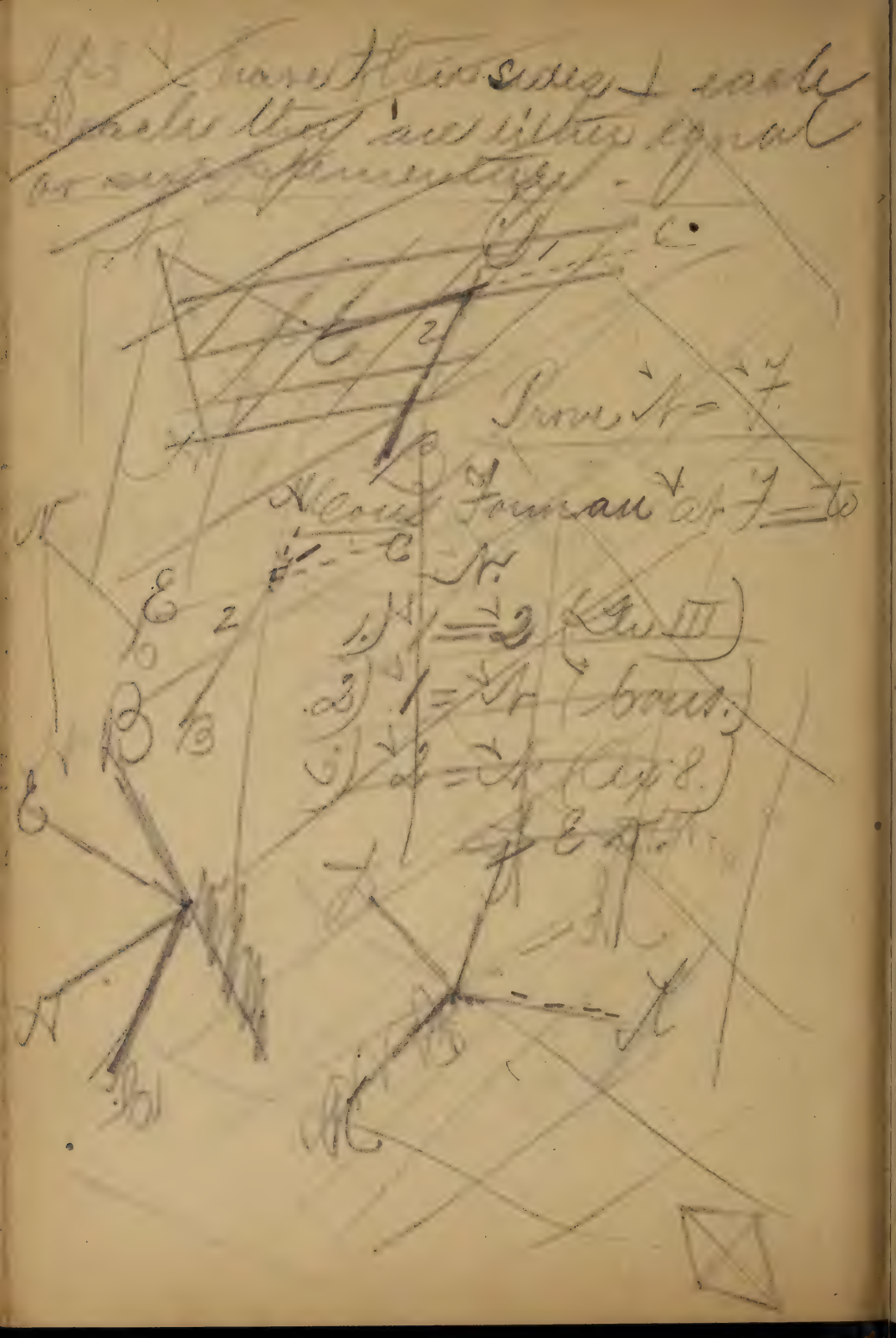
Prove EB the only line in the drawn $\triangle Ab$ from E.

$$\angle ACB = \angle ACB + \angle BCA - \angle BCA$$

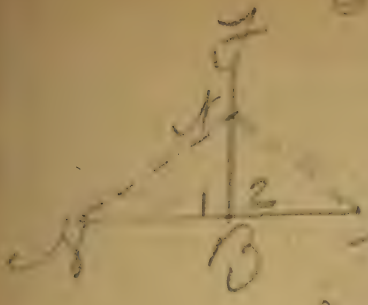
Orig. View Broadway
Looking North from Liberty







Q. 34



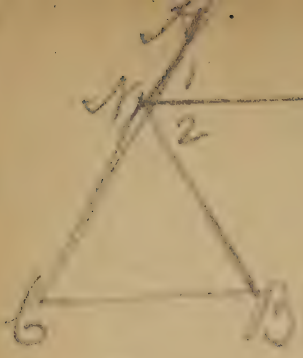
Prove $AA = AC$

1) $\overline{AA} = \overline{AB}$ = 1 RC (Given)
2) $AA = AC$ (Angles)
3) Alt ∠s converse
4) $\overline{AA} \parallel \overline{AB} = \overline{BC}$ (Alt ∠s ≅)
5) $\overline{AA} \parallel \overline{AC}$

$\overline{AA} = \overline{AC}$.

Q.E.D.

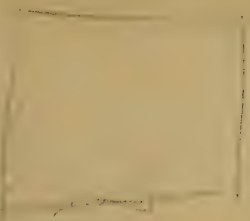
If at the midpoint of a line segment a perpendicular is drawn, then any point on the perpendicular is equidistant from the endpoints of the line segment.



№ 88

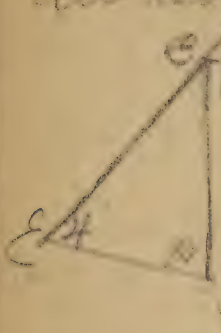
1) $\angle A + \angle B = \angle B + \angle B$ (A ∥ Б)
2) $\angle A = \angle 1 + \angle 2$ (Ч. 2)
3) $\angle B + \angle B = \angle 1 + \angle 2$ (Ч. 3)
4) $\angle B = \angle 2$ (Ч. 2)
5) $\angle 1 = \angle 2$ (Ч. 3)
6) $\angle B = \angle 1$
7) $AC \parallel BB$ (Ч. 6)

Ч. Т. Д.



Cu

The bisectors of the ext ∠s through of
cu meet ⊙ at to have

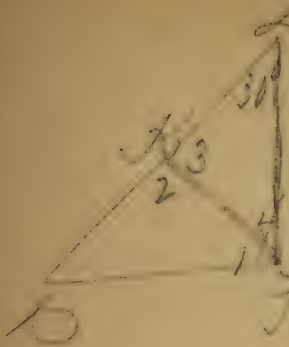


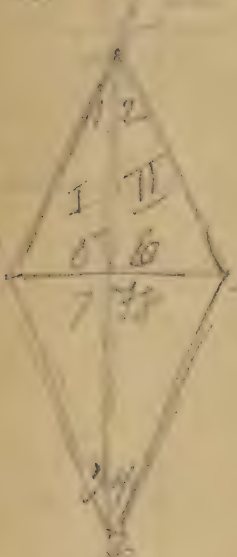
Typ 2

1) ∠HB7 = ∠4 + ∠2 (∠th ∠s of 6)
2) ∠4 = ∠3 (∠s in ⊙)
3) ∠3 = ∠1 (Alt ∠s)
4) ∠1 = ∠2
5) BC ∥ HE (∠s ∥ EJ (∠th 6 ≅)

∴ ∠ ∥ ∠s

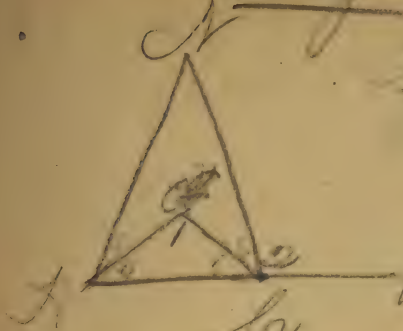
∥ ∠4 + ∠2 = ∠HB7 (∠s of
4) ∠3 + ∠1 = ∠HB7
3)


30
3
2
4
1


I
II

Original 80.

Hyp: $\triangle ABC$ with $\overline{AB}$ extended to D

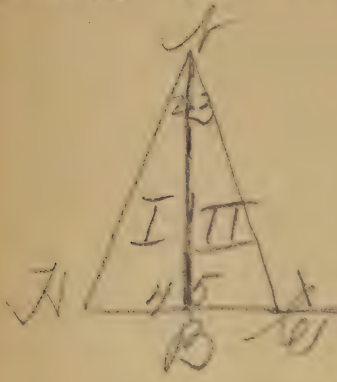


Prove $\angle 1 = \angle 2$

1)	$\angle 3 = \angle A$	(Hyp.)
2)	$\angle 6 = \angle C$	(Hyp.)
3)	$\angle 3 + \angle 6 = \angle A + \angle C$	(Ax. 1.)
4)	$\angle A + \angle C + \angle 1 = 2$ Rt $\angle$s	(Sum $\angle$s)
5)	$\angle 3 + \angle 6 + \angle 2 = 2$ Rt $\angle$s	(Sum $\angle$s)
6)	$\angle 1 = \angle 2$	(Ax. 8.)

Q. E. D.

Euclid II. 2 & 4



Ag. a line $\triangle ABC$ with vertex A.

Prove $\triangle = \frac{1}{2} BC \cdot h$

Through A draw $AD \parallel BC$

Let $AD = x$

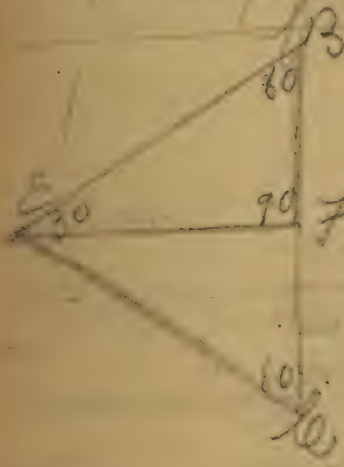
Then $\triangle = \frac{1}{2} x \cdot h$

Draw $AE \perp BC$

Then $\triangle = \frac{1}{2} BC \cdot AE

Aug 26

If a ∠ of a Δ is equal to two ∠s of another Δ respectively, and the sides containing the equal ∠s are proportional, the Δs are similar. Also, corresponding sides of similar Δs are proportional, and the ∠s opposite the homologous sides are equal.



$\angle B = \angle S$

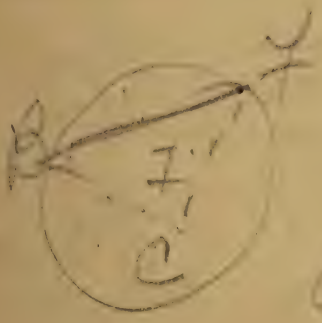
$\angle C = \angle S$

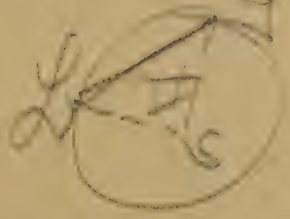
$BF = \angle J$

$\triangle I = \triangle II$

$\angle C = \angle S$

$BF_1 = \angle J$



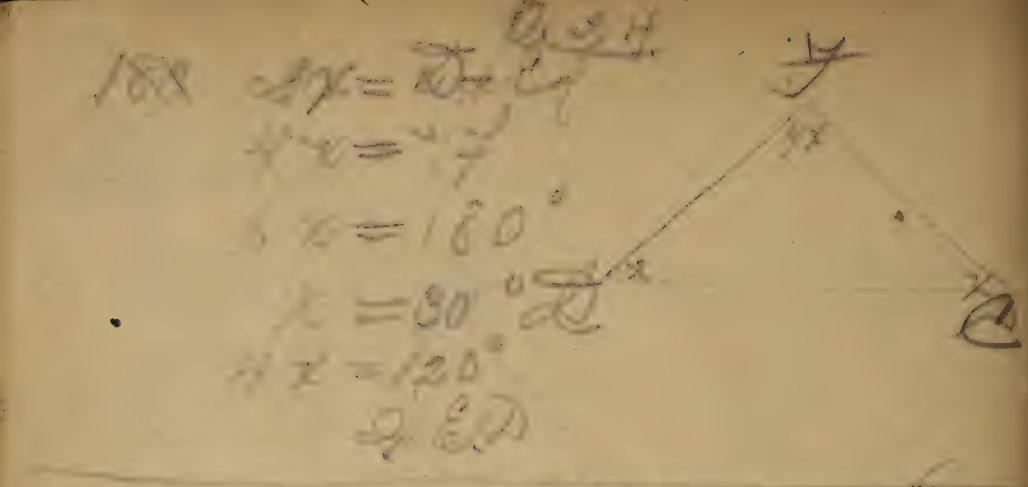


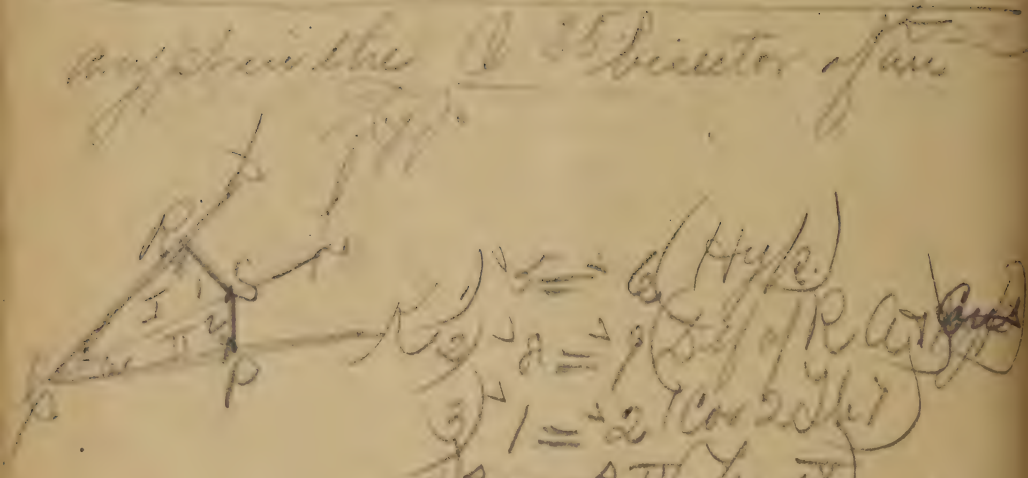
Draw a scalene $\triangle$ triangle & prove formula for the interior of the acute angles of a triangle of 135°.

Part E.

$1 = 2$
$3 = 4$
$1+3 = 2+4$

$x = 3 + 4$
$y = 2 + 4$
$90° + 2x + 2y = 180°$ Sum ∠s of △
$135° + x + y = 180°$ (Ax 3)
$x + y = 45°$ (Ax 3)

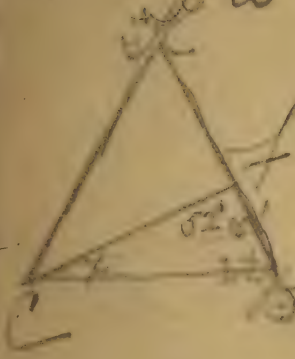




Corresponding parts of congruent Δs are equal.

RS bisects ∠PRQ.

The bisector of two exterior angles of a triangle make with the third side an angle of $52^\circ 20'$. Find the other angles of the triangle.



Find $\angle E$, $\angle F$, $\angle CDE$.

$180^\circ - 52^\circ 20' = 127^\circ 40'$

$\angle C + \angle F = 127^\circ 40'$

$\angle C = \angle F$

$2x + 127.40 = 180^\circ$

$3x + 52^\circ 20' = 180^\circ$

$3x = 127^\circ 40'$

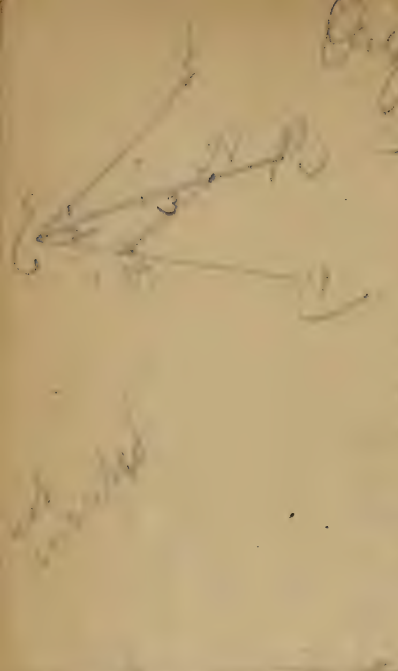
$x = 42^\circ 33'$

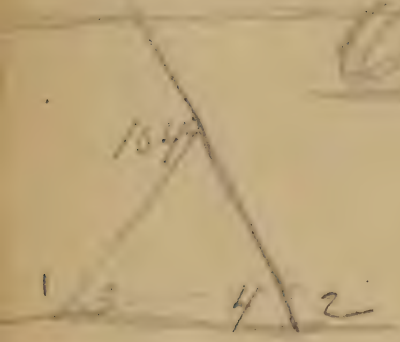
$2x = 85^\circ 20'$

$\angle E = 85^\circ 20'$

$\angle F = 42^\circ 33'$

$\angle C = 42^\circ 40'$





Book 20.

Prop 1. In a parallelogram the diagonal makes with one side an angle equal to the angle which the other side makes with the other.

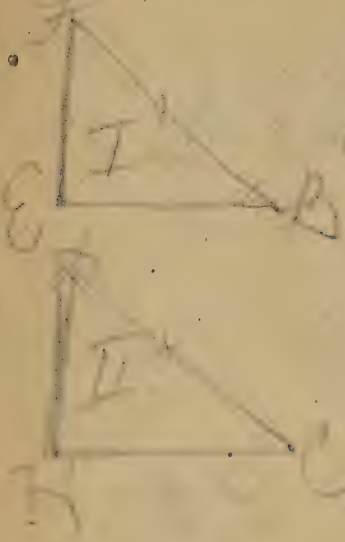
not corrected

1) $\angle 1 = \angle 2$ (Alt. $\angle$s)
2) $\angle 3 = \angle 4$ (Alt. $\angle$s)
3) $\triangle ABC = \triangle ADC$ (Cor. 2 Prop. 1)
4) $\angle C = \angle A$ (Eq. 1, 2, 3)
5) $\angle I = \angle II$ (Ax. 1, Eq. 4)

Q. E. D.

Original Ip. 215

If 2 ∠s of Δ are ≅ to 2 ∠s of another Δ & one side of 1st Δ is ≅ to corresponding side of 2nd Δ, then the Δs are ≅ in all respects.



Hyp. $\angle B = \angle C$

$\angle B = \angle C$

Prove $\triangle I \cong \triangle II$.

1) $\angle E = \angle H$ (Ax 10)
2) $\angle B = \angle C$ (Hyp)
3) $\angle I = \angle D$ (Cor 2 Th 4)
4) $EB = HC$ (Th II)
5) $\triangle I = \triangle II$ (Th III)

Q.E.D.

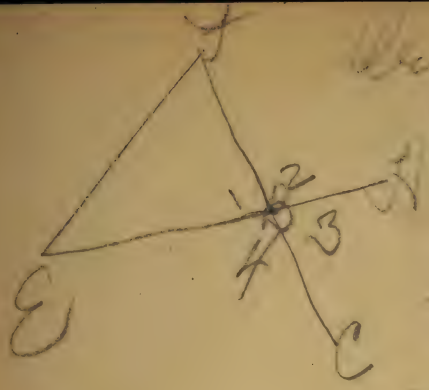
Converse

Hyp. $\angle A < \angle K$

$\angle B > \angle K$

Prove $AB < KB$

1) $\angle A + \angle B > \angle K + \angle B$ (Ax 9)
2) $\angle C > \angle K - \angle B$ (Ax 3)
3) $AB < KB$ (Th 19)



Дано ∠1 = ∠2

∠1 = ∠E + ∠7
∠2 = ∠E + ∠7 } ∠1 и ∠2 ∠ 6.
∠1 = ∠2 (ч.т.д.)
ЧТД

Упр 16. 6 рис Вид

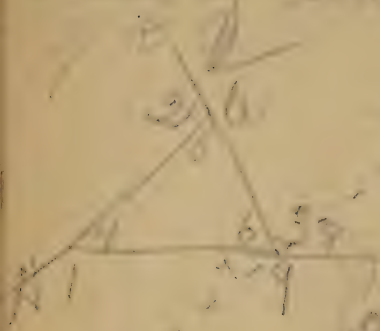
Дано ∠ ... + ∠ ...

∠ ... > ∠ ... − ∠ ...

1) ∠ ... и ∠ ... смежные
2) ∠ ... + ∠ ... = а развернутый
3) ∠ ... < ∠ ... + ∠ ... (1 и 2)

4) ∠ ... + ∠ ... > ∠ ...
5) ∠ ... > ∠ ... − ∠ ...

Original Th. XII. D. 13



Prove $\angle 1 + \angle 2 + \angle 3 = 4$ Rt $\angle$s

1) $\angle 2 = \angle 4 + \angle 6$ (Ext $\angle$ Thm)
2) $\angle 1 = \angle 5 + \angle 6$ (" " ")
3) $\angle 3 = \angle 4 + \angle 5$ (" " ")
4) $\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + 2\angle 6 + \angle 4 + \angle 5$
5) $\angle 4 + \angle 5 + \angle 6 = 2$ Rt $\angle$s.
6) $\angle 1 + \angle 2 + \angle 3 = 4$ Rt $\angle$s (Ewts)

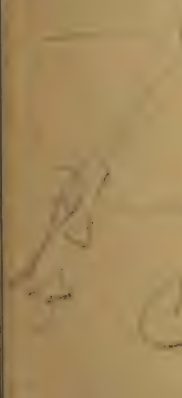
If two equal sides of an isos. Δ be produced, then the exterior angles formed are equal.

Hyp. $\overline{AB} \cong \overline{AC}$

Prove $\angle 3 \cong \angle 4$

1)	$\angle 1 \cong \angle 2$	(Vert. $\angle$s)
2)	$\angle 1 + \angle 3 = 2$ rt. $\angle$s	(Def. I)
3)	$\angle 2 + \angle 4 = 2$ rt. $\angle$s	(Def. I)
4)	$\angle 3 = 2$ rt. $\angle$s $- \angle 1$	(Ax. 2)
5)	$\angle 4 = 2$ rt. $\angle$s $- \angle 2$	(Subst. in Eq. 4)
6)	$\angle 3 \cong \angle 4$	(Ax. 9)

Q.E.D.

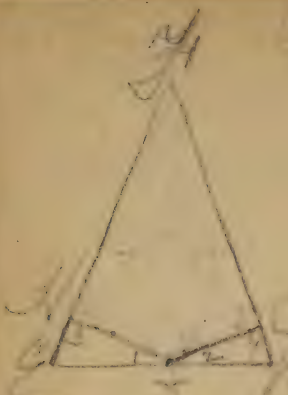


Prove $\angle 3 \cong \angle 4$

1)	$\angle 1 \cong \angle 2$	(Vert. $\angle$s)
2)	$\angle 1 + \angle 3 = 2$ rt. $\angle$s	(Def. I)
3)	$\angle 2 + \angle 4 = 2$ rt. $\angle$s	(Def. I)
4)	$\angle 3 = 2$ rt. $\angle$s $- \angle 1$	(Ax. 2)
5)	$\angle 4 = 2$ rt. $\angle$s $- \angle 2$	(Subst. in Eq. 4)
6)	$\angle 3 \cong \angle 4$	(Ax. 9)

Q.E.D.

Bog 14 D.11



Hyp. In $\triangle EFB$ with $\perp$ drop from E to side $\overline{FB}$

Prove. $\angle 1 \cong \angle 2$

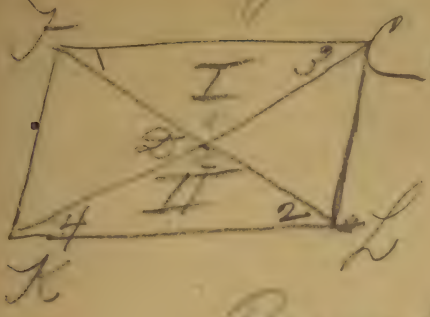
1) $\angle E \cong \angle B$ (Thm X)
2) $\angle 3 \cong \angle 4$ (Hyp.)
3) $\therefore \angle 1 \cong \angle 2$ (Sum of $\angle$s Cor.)

Q E D

Proved.
By means of Compass
and Ruler & Known Euclidean
Proved.

XXX

Original 13.



Hyp. $\angle 7 \cong \angle L$, $\angle 4 \cong \angle 2$

Diag. $7K \cong KL$

Prove $\triangle I = \triangle II$

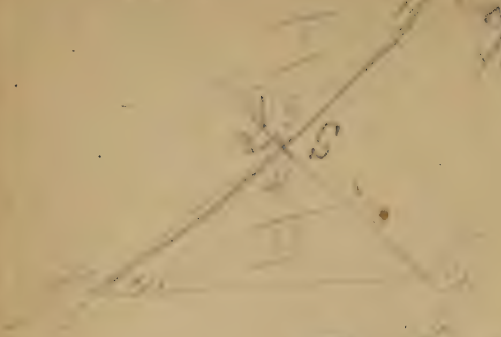
1) $\triangle 1 = \triangle 2$
2) $\triangle 3 = \triangle 4$ ($\angle$s $\cong$ =)
3) $7C = KL$ (Hyp.)
4) $\triangle I = \triangle II$ (Th. IX.)

Q. E. D.

Original 14.

If from any pt. on the face of an isos. $\triangle$ I's be dropped upon the 2 eq. sides they will make equal angles with the base.

Exercise 12 p. 8



Hyp: $\overline{FC} \parallel \overline{AE}$
$\overline{AC}$ bisects $\overline{FE}$
Prove: $\triangle I \cong \triangle II$

Proof:

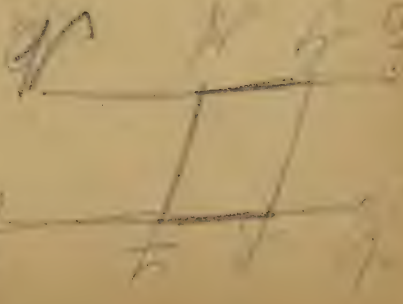
1) $\overline{FI} \cong \overline{IE}$ (Def. bisect)
2) $\overline{AI} \cong \overline{IC}$ (Def. bisect)
3) $\angle AIE \cong \angle CIF$ (Vert. $\angle$s)
4) $\triangle I \cong \triangle II$ (SAS Th. IV)
5) Q.E.D.

Доказать $\triangle I = \triangle II$

1) $\angle K = \angle P$ (накрест лежащие)
2) $AK = AP$
3) $\angle 1 = \angle 2$ (накрест лежащие)
4) $\triangle I = \triangle II$

Что и требовалось доказать.

Дано: $MK \parallel NP$, $MN \parallel KP$



Chap 10. Sec 6

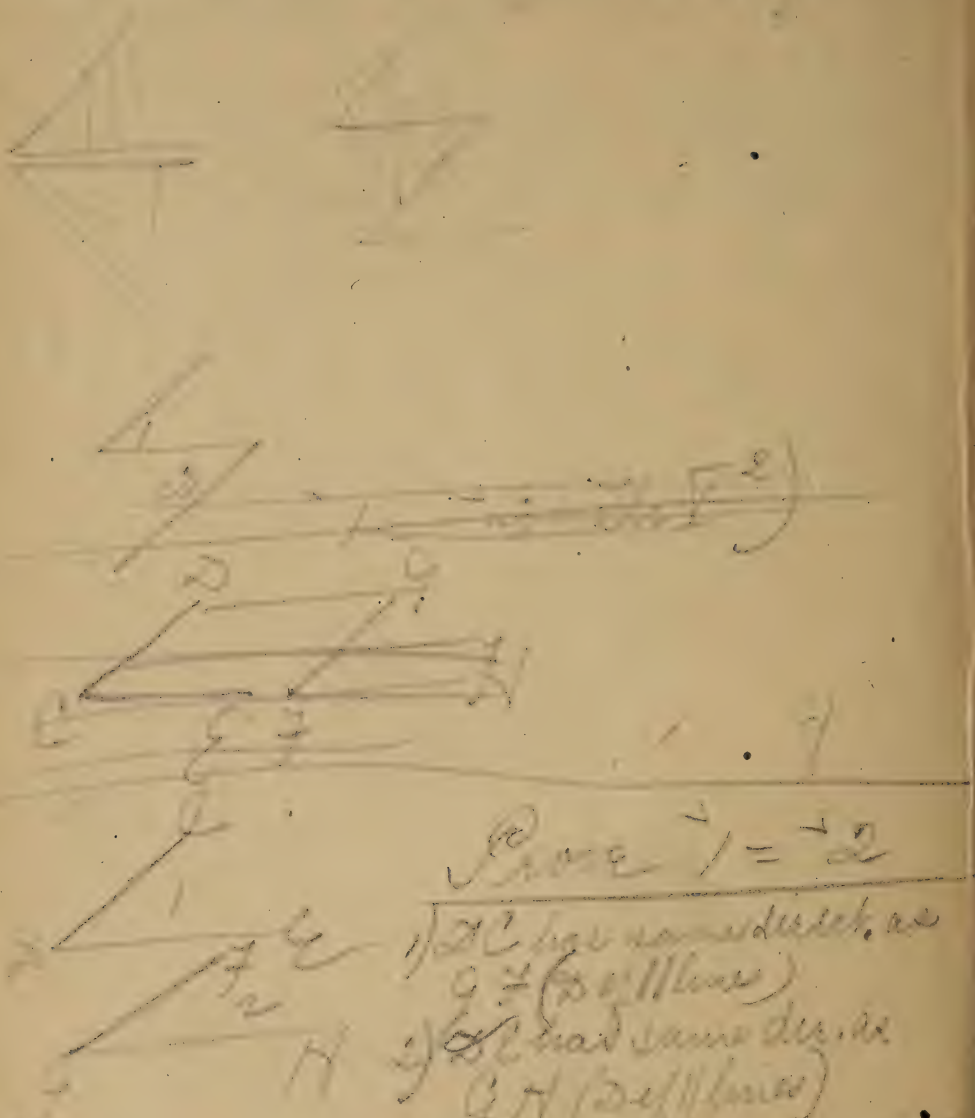
Prove $\angle I = \angle II$

1) Line $CG = \overline{IV}$ (All lines)
2) Let $\angle$ [illegible]
3) $\angle = \angle$ ($\angle$s I & II)
4) $\angle I = \angle II$ ($\angle$s III)

Or

[illegible]

Better



Prove $\angle 1 = \angle 2$

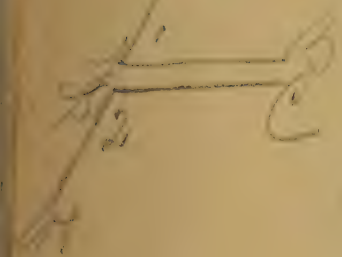
1) $\overline{DC}$ has no direct. as $\overline{GH}$ (Both $\parallel$ lines)

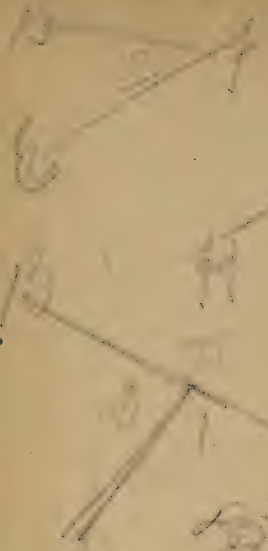
2) $\overline{DC}$ had same dir. as $\overline{GH}$ (Both $\parallel$ lines)

3) And $\overline{DC}$ & $\overline{DE}$ as $\overline{GH}$ & $\overline{GF}$ are diff dir. below $\overline{GF}$ & $\overline{GH}$

$\therefore$ 4) $\angle 1 = \angle 2$ (Corr. $\angle$s of $\parallel$ lines)

Two angles two of whose sides are || and lying in the same direction from their vertices & the other two sides are || and lying in the directions from their vertices, are supplements of each other





$\angle 4 \cong \angle 2$ $133 = 2 + 2$

$\angle 5 \cong \angle 3$ $80 = 2 + 2$

$\angle 2 + \angle 3 = 180°$ $80 - 80 = 100$

$\angle 2 = 100°$

$180 - 133 = 47$

$\angle 3 = 47$

47

147

180

147

$33°$

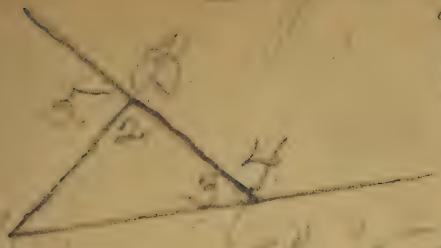
$\angle 3 \cong \angle BCD = 100°$ ——— $2 + 3$

$133 - 80$

47

$\angle 5 = 133 = 2 + 100°$

$\angle 4 = 80 = 4 +$

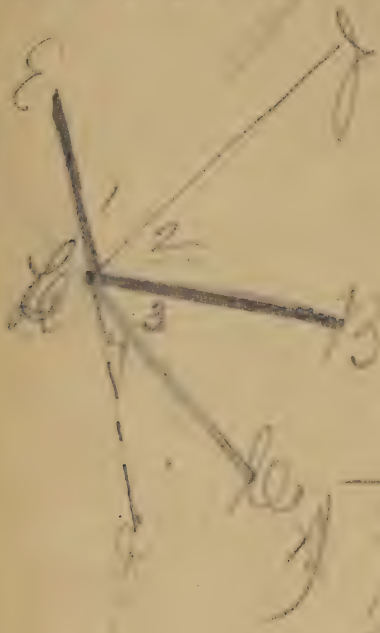


$\angle 3 = \angle 4 + 100°$

$\angle 3 = 80 + 20°$

$\angle 3 = 100°$

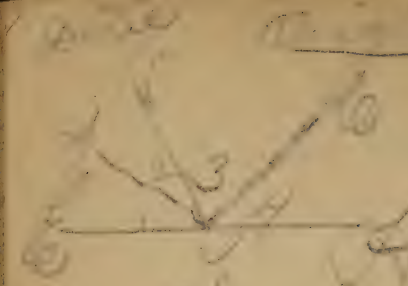
Ex angles 2 & 3 if two sides are ∥ & both in same direction from vertex & two sides are ∥ but in opp dir from vertex are sup of each other


E
1
2
3
B
C

$\angle ABC = 180^\circ$

$\angle ABY = 160^\circ$

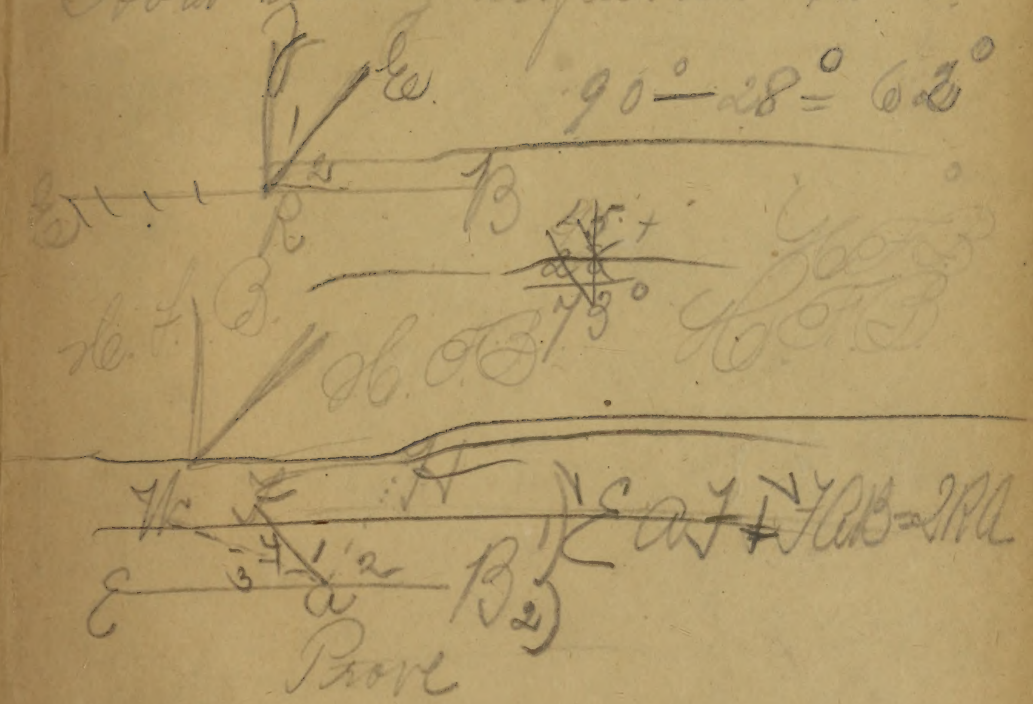
$\angle XBY = 20^\circ$



Practical examples

1. 2 ∠s are complementary and the greater exceeds the lesser by 28 degrees. How many degrees in each?

2. Two angles are supplementary & the greater is 8 times the lesser. How many degrees in each.

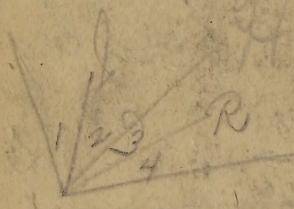


Prove

Euclid I.

Dan

If any angle be bisected into 2 angles and each of these angles be bisected the angle between the bisectors will be the original angle.



Hyp. ∠EFB divided into 2 angles ∠EFH & ∠HFB and ∠EF bisected forming ∠EFR & ∠RFB

Trans. ∠FR = ½∠EFB. ∠FH = ∠EFH + ∠HFB

bisected forming ∠HFR = ∠RFB

1) ∠1 = ∠2 } Ax 6
2) ∠3 = ∠4 }

3) ∠1 + ∠3 = ∠2 + ∠4 (Ax 2)

4) ∠2 + ∠3 = ∠EFR (Ax 2)

5) ∠1 + ∠4 = ∠RFB (Ax 2)

6) ∠EFR = ½∠EFB (Ax 7)

7) ∠1 + ∠3 = ½∠EFB (Ax 8)

8) ∠EFR = ∠RFB (Ax 8)

9) ∠EFR = ½∠EFB (Ax 4)

Q.E.D.

C

Euclid I. 13